# Numerical solution of matrix equations arising in control of bilinear and stochastic systems

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## Abstract

Many system-theoretic computations, like the (stability) analysis of linear state-space systems or model reduction of such systems via balanced truncation, require the solution of certain linear or nonlinear matrix equations. In the linear case, these are Lyapunov or algebraic Riccati equations.

In previous years, we have investigated bilinear and stochastic linear systems. Again, in their stability analysis as well as in model reduction by balanced truncation, linear and nonlinear matrix equations arise that have to be solved numerically. Primarily, we will discuss the generalized *linear* matrix equations associated to bilinear and stochastic control systems, where in addition to the Lyapunov operator, a positive operator appears in the formulation of the equations. We will provide some results in the spirit of Lyapunov and inverse Lyapunov theorems in the spirit of Hans Schneider's work of 1965, relating properties of the solution to these matrix equations to stability of stochastic systems.

Furthermore, we investigate the numerical solution of these *Lyapunov-plus-positive equations*. Due to the large-scale nature of these equations in the context of model order reduction, we study possible low rank solution methods for them. We show that under certain assumptions one can expect a strong singular value decay in the solution matrix allowing for low rank approximations. We further provide some reasonable extensions of some of the most frequently used linear low rank solution techniques such as the alternating directions implicit (ADI) iteration and the extended Krylov subspace method. By means of some standard numerical examples used in the area of bilinear model order reduction, we will show the efficiency of the new methods.

Time permitting, we will briefly touch upon extensions to a special class of Sylvester equations also related to model reduction of bilinear systems, but also appearing in fitting algorithms for smooth kernels in image reconstruction, as well as certain nonlinear matrix equations arising in model reduction for stochastic systems.

## Keywords

Matrix equations, Bilinear systems, Stochastic systems, Numerical algorithms.

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