The likelihood ratio test in bilinear models

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Abstract

Let W_{H_0} and W_{H_1} be two independently distributed Wishart matrices which build up Wilks Λ , i.e.

$$\Lambda = \frac{|W_{H_0} + W_{H_1}|}{|W_{H_1}|}.$$

The matrices appear when testing H_0 : BG = 0 versus H_1 : B unrestricted in a MANOVA model, i.e.

$$X = BC + E,$$

X is a random matrix which represents the observations, C and G are known matrices, and $E \sim N_{p,n}(0, \Sigma, I)$, where B and Σ are unknown parameter matrices. The distribution of Λ equals a product of independent beta-distributed variables. When approximating the distribution several approaches are available, where the most commonly applied uses approximations of the gamma-function.

Let the GMANOVA model be given by

$$X = ABC + E,$$

where in addition to the MANOVA model a known matrix A has been introduced.

Remarkable is an old classical result which states that the likelihood ratio test for testing in a GMANOVA model H_0 : FBG = 0, where F and G are known, versus H_1 : B unrestricted also follows a Wilks Λ distribution.

It is remarkable since the maximum likelihood estimators in the MANOVA and GMANOVA are very different. The talk will derive the distribution in a somewhat different way than what usually is applied which also sheds some light on some conditional arguments.

Keywords

GMANOVA, Growth curve model, Likelihood ratio test.