# Joint numerical range and numerical shadow

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## Abstract

Numerical range of a matrix $X$ of order $N$ can be interpreted as a projection of the set of mixed quantum states of size $N$ onto a plane determined by $X$. We show that for a random Ginibre matrix $G$ with spectrum asymptotically confined in the unit disk, its numerical range forms a disk of radius $\sqrt{2}$. This result is shown to be related to the Dvoretzky theorem. Numerical shadow $P_X(z)$ of an operator $X$ is the probability measure on the complex plane supported by the numerical range $W(X)$, defined as the probability that the inner product $(Xu, u)$ is equal to $z$, where u denotes a normalized N-dimensional random complex vector. Restricting vectors u to a certain subset of the set of all states (e.g. real/product/entangled states) one arrives at the notion of the restricted numerical range, which in general needs not to be convex. Analyzing numerical shadow of hermitian matrices with respect to real states we show that they form a generalization of the standard B-spline. We analyze also joint numerical range of a triple of hermitian operators which can be related with a 3D convex body.

## Keywords

Numerical range, Numerical shadow, Nonhermtian random matrices.

## References

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