COIMBRA · PORTUGAL

7 – 11 September

MAT TRIAD 2015

Conference on Matrix Analysis and its Applications

BOOK OF ABSTRACTS

Department of Mathematics,
University of Coimbra, Portugal
September 7–11, 2015

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Part I

Information

Information

The MAT-TRIAD 2015 - *Conference on Matrix Analysis and its Applications* will be held in the Department of Mathematics, University of Coimbra, Portugal, from 7 to 11 September 2015, continuing the tradition of MAT-TRIAD conferences organized in a biannual format since 2005.

The purpose of the conference is to bring together researchers sharing an interest in a variety of aspects of matrix analysis and its applications, special emphasis being given to applications in other areas of science. One of the main goals is to highlight recent achievements in these mathematical domains. The programme will cover different aspects with emphasis on recent developments in matrix and operator theory, direct and inverse spectral problems, matrices and graphs, applications of linear algebra in statistics, matrix models in industry and sciences, linear systems and control theory, quantum computation and combinatorial matrix theory. The conference will provide a friendly atmosphere for the discussion and exchange of ideas, which hopefully will lead to new scientific links among participants.

The scientific programme of this meeting will involve plenary talks, invited mini-symposia, sessions of contributed talks and a poster presentation. Two short courses delivered by leading experts will also occur. The invited lecturers of these two short courses are:

Friedrich Pukelsheim (Germany),
Peter Šemrl (Slovenia).

The invited speakers are:

Peter Benner (Germany),
Marija Dodig (Portugal),
Froilán M. Dopico (Spain),
Moshe Goldberg (Israel),
Christian Mehl (Germany),
Dietrich von Rosen (Sweden),
Roman Zmyślony (Poland),
Karol Życzkowski (Poland),

as well as the winners of Young Scientists Awards of MAT-TRIAD 2013 which took place at Herceg Novi, Montenegro:

Maja Nedović (Serbia),
Jaroslav Horáček (Czech Republic).

The following thematic invited mini-symposia will occur:

IMS1. *Linearizations and ℓ-ifications of Matrix Polynomials: theory and applications*, organized by Maribel Isabel Bueno (USA), Froilán Dopico (Spain) and Susana Furtado (Portugal);

IMS2. *Spectral Graph Theory*, organized by Domingos M. Cardoso (Portugal);

IMS3. *Algebraic Methods in Operator Theory*, organized by M. Cristina Câmara (Portugal);

IMS4. *Coding Theory*, organized by Raquel Pinto and Diego Napp (Portugal);

IMS5. *Matrix Theory, Applications and Engineering*, organized by Marko Stošić (Portugal);

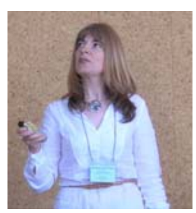
IMS6. *Functions of Matrices*, organized by Pedro Freitas and Sónia Carvalho (Portugal);

IMS7. *Linear Preserver Problems*, organized by Henrique F. da Cruz and Rosário Fernandes (Portugal);

IMS8. *Statistical Inference, Numerical and Combinatorial Methods*, organized by Luís Miguel Grilo and Fernando Lucas Carapau (Portugal);

IMS9. *Statistical Models with Matrix Structure*, organized by Miguel Fonseca (Portugal).

There will be a special session in memoria of our dearest colleague Glória Cravo.



To end these five days conference, a talk delivered by J. Vitória (Portugal) will focus on "Linear Algebra in Portugal".

The work of young scientists will receive special consideration in MAT-TRIAD 2015, following the example of the previous MAT-TRIAD meetings. The best talk of graduate student or scientist having recently completed a Ph.D. will be awarded. Prize-winning works will be widely publicized and promoted by the conference.

The number of registered participants is around 160.

A special issue dedicated to MAT-TRIAD 2015 with papers of participants will be published by Springer Verlag after refereing procedure in a volume entitled *Applied and Computational Matrix Analysis* in the series *Proceedings of Mathematics & Statistics*.

The social programme includes:

- an excursion to the *Forest and Royal Palace* of Bussaco,
- the conference dinner at Loggia Museu Nacional Machado Castro, including a 40 minutes concert played by the group *Quarteto Santa Cruz de Coimbra* and Joana Neto. Concert program:
  - PUCELL - SUITE Abdelazer
  - VIVALDI - CESSATE, OMAI CESSATI
  - HAENDEL LASCIA CHIO PIANGA

  Chant: Joana Neto
  Violins: António Ramos and Clara Dias
  Violet: Ricardo Mateus
  Violoncello: Rogério Peixinho
  The banket speaker will be Francisco Carvalho.
- a visit to the "Paço das Escolas" of the University of Coimbra.

Updated information is available at http://www.mattriad.ipt.pt

Part II

Invited Speakers

Numerical solution of matrix equations arising in control of bilinear and stochastic systems

Peter Benner

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Abstract

Many system-theoretic computations, like the (stability) analysis of linear state-space systems or model reduction of such systems via balanced truncation, require the solution of certain linear or nonlinear matrix equations. In the linear case, these are Lyapunov or algebraic Riccati equations.

In previous years, we have investigated bilinear and stochastic linear systems. Again, in their stability analysis as well as in model reduction by balanced truncation, linear and nonlinear matrix equations arise that have to be solved numerically. Primarily, we will discuss the generalized *linear* matrix equations associated to bilinear and stochastic control systems, where in addition to the Lyapunov operator, a positive operator appears in the formulation of the equations. We will provide some results in the spirit of Lyapunov and inverse Lyapunov theorems in the spirit of Hans Schneider's work of 1965, relating properties of the solution to these matrix equations to stability of stochastic systems.

Furthermore, we investigate the numerical solution of these *Lyapunov-plus-positive equations*. Due to the large-scale nature of these equations in the context of model order reduction, we study possible low rank solution methods for them. We show that under certain assumptions one can expect a strong singular value decay in the solution matrix allowing for low rank approximations. We further provide some reasonable extensions of some of the most frequently used linear low rank solution techniques such as the alternating directions implicit (ADI) iteration and the extended Krylov subspace method. By means of some standard numerical examples used in the area of bilinear model order reduction, we will show the efficiency of the new methods.

Time permitting, we will briefly touch upon extensions to a special class of Sylvester equations also related to model reduction of bilinear systems, but also appearing in fitting algorithms for smooth kernels in image reconstruction, as well as certain nonlinear matrix equations arising in model reduction for stochastic systems.

Keywords: matrix equations, bilinear systems, stochastic systems, numerical algorithms.

References

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Matrix pencils completions, combinatorics, and integer partitions

Marija Dodig

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Abstract

The general Matrix Pencil Completion Problem (MPCP), apart of purely theoretical importance, has important motivations and applications in control theory of linear systems, including pole placement, non-regular feedback, dynamic feedback, zero placement and early-stage design. Due to the complexity of the problem, although studied by many authors, it still remains open. In the recent years new combinatorial methods have appeared that involve novel objects and tools related to combinatorial properties and comparisons of partitions of integers. We shall present some of the most important results involving combinatorial results on partitions of integers, and some mile-stones towards a solution of MPCP.

Inverse eigenstructure problems for matrix polynomials

Froilán M. Dopico[1], Fernando De Terán[1], D. Steven Mackey[2] and Paul Van Dooren[3]

[1]Departamento de Matemáticas, Universidad Carlos III de Madrid, Leganés, Spain
[2]Department of Mathematics, Western Michigan University, Kalamazoo, Michigan, USA
[3]Department of Mathematical Engineering (INMA/ICTEAM), Université Catholique de Louvain, Louvain-la-Neuve, Belgium

Abstract

In this talk, we summarize several results on inverse eigenstructure problems for matrix polynomials that have been obtained recently in [Terán et al., to appear] and [Terán et al., submitted], and discuss how they complete other results previously known in the literature. Three key features of these new results are that they are valid for singular matrix polynomials, they consider prescribed minimal indices, in contrast to many inverse results in the literature which only deal with prescribed elementary divisors, and that certain degrees are also prescribed. In particular, we present necessary and sufficient conditions for the existence of a matrix polynomial when its degree, its finite and infinite elementary divisors, and its left and right minimal indices are prescribed, and necessary and sufficient conditions for the existence of dual minimal bases with prescribed row-degrees. In both cases, these necessary and sufficient conditions are determined mainly by the so called "index sum theorem". In addition, the solutions we present of the inverse problems mentioned above are constructive and are based on a new class of sparse, structured matrix polynomials that we have baptized as polynomial zigzag matrices.

Keywords: matrix polynomials, minimal indices, minimal bases, inverse problems.

References

Terán, F. De, Dopico, F. M. and Van Dooren, P. *Matrix polynomials with completely prescribed eigenstructure*, to appear in SIAM Journal on Matrix Analysis and Applications.

Terán, F. De, Dopico, F. M., Mackey, D. S. and Van Dooren, P. *Polynomial zigzag matrices, dual minimal bases, and the realization of completely singular polynomials*, submitted.

Radii of elements in finite-dimensional power-associative algebras

Moshe Goldberg

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Abstract

In this talk we extend the notion of spectral radius to elements in arbitrary finite-dimensional power-associative algebras over the real or complex numbers. Time allowing, we shall illustrate the new concept by resorting to three related topics: a variant of the Gelfand formula, stability of subnorms, and the functional power equation.

Generic low rank perturbations of structured matrices

Christian Mehl

Technische Universität Berlin, Institut für Mathematik, Germany

Abstract

The effect of generic low rank perturbations on the Jordan structure of general matrices is well known. If the perturbation matrix has rank k, then for each eigenvalue the largest k Jordan blocks associated that eigenvalue will disappear while all other Jordan blocks associated with that eigenvalue will remain. Surprisingly, this behavior changes if generic structure-preserving perturbations are applied to matrices that have symmetry structures with respect to some indefinite inner product. Important examples include J-Hamiltonian matrices, i.e., real or complex $2n \times 2n$ matrices A that satisfy $A^*J + JA = 0$ for some invertible skew-symmetric matrix J. For such matrices it has been observed that sometimes Jordan blocks may generically grow in size after perturbation.

In this talk, we give an explanation for this surprising behavior by giving an overview over the theory of generic structure-preserving low rank perturbations of structured matrices. While the first part of the talk focusses on rank-one perturbations, the second part considers the case of perturbations of arbitrary rank k.

The talk is based on joined work with Leonhard Batzke, Volker Mehrmann, André C.M. Ran, and Leiba Rodman and is dedicated to the memory of Leiba Rodman.

Keywords: Hamiltonian matrices, perturbations theory, low rank perturbations, generic perturbations.

The likelihood ratio test in bilinear models

Dietrich von Rosen[1,2]

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[2]MAI, Linköping University, Sweden

Abstract

Let W_{H_0} and W_{H_1} be two independently distributed Wishart matrices which build up Wilks Λ, i.e.

$$\Lambda = \frac{|W_{H_0} + W_{H_1}|}{|W_{H_1}|}.$$

The matrices appear when testing H_0: $BG = 0$ versus H_1: B unrestricted in a MANOVA model, i.e.

$$X = BC + E,$$

X is a random matrix which represents the observations, C and G are known matrices, and $E \sim N_{p,n}(0, \Sigma, I)$, where B and Σ are unknown parameter matrices. The distribution of Λ equals a product of independent beta-distributed variables. When approximating the distribution several approaches are available, where the most commonly applied uses approximations of the gamma-function.

Let the GMANOVA model be given by

$$X = ABC + E,$$

where in addition to the MANOVA model a known matrix A has been introduced.

Remarkable is an old classical result which states that the likelihood ratio test for testing in a GMANOVA model H_0: $FBG = 0$, where F and G are known, versus H_1: B unrestricted also follows a Wilks Λ distribution.

It is remarkable since the maximum likelihood estimators in the MANOVA and GMANOVA are very different. The talk will derive the distribution in a somewhat different way than what usually is applied which also sheds some light on some conditional arguments.

Keywords: GMANOVA, growth curve model, likelihood ratio test.

Inference in linear mixed models and Jordan algebra

Roman Zmyślony

University of Zielona Góra

Abstract

This presentation will show usefulness of Jordan algebra in estimation and testing hypotheses in linear mixed models. In fact, the good properties of estimators and tests, will be explain in terms of Jordan algebras. The linear models and its inference will be explain in coordinate free approach. Namely, existence of BLUE for parametric estimable functions will be given in explicit form, and the test statistic for testing hypotheses about single parameter will be function of unbiased estimators. In the case when BLUE and BQUE exist for all parameters of fixed effects and covariance matrix, respectively, under additional assumption of normality, the estimators are BUE, because they are functions of complete sufficient statistics. Moreover, the distribution function of test statistics will be given. This idea can be applied for multivariate linear models, which will be presented in lecture given by Kozioł.

Keywords: free coordinate approach, Jordan algebra, mixed linear models, unbiased estimation, testing hypotheses.

References

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Joint numerical range and numerical shadow

Karol Życzkowski[1,2]

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[2]Center for Theoretical Physics, Polish Academy of Sciences, Warsaw, Poland

Abstract

Numerical range of a matrix X of order N can be interpreted as a projection of the set of mixed quantum states of size N onto a plane determined by X. We show that for a random Ginibre matrix G with spectrum asymptotically confined in the unit disk, its numerical range forms a disk of radius $\sqrt{2}$. This result is shown to be related to the Dvoretzky theorem. Numerical shadow $P_X(z)$ of an operator X is the probability measure on the complex plane supported by the numerical range $W(X)$, defined as the probability that the inner product (Xu, u) is equal to z, where u denotes a normalized N-dimensional random complex vector. Restricting vectors u to a certain subset of the set of all states (e.g. real/product/entangled states) one arrives at the notion of the restricted numerical range, which in general needs not to be convex. Analyzing numerical shadow of hermitian matrices with respect to real states we show that they form a generalization of the standard B-spline. We analyze also joint numerical range of a triple of hermitian operators which can be related with a 3D convex body.

Keywords: numerical range, numerical shadow, nonhermtian random matrices.

References

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Dunkl, C. F., Gawron, P., Pawela, L., Puchała, Z. and Życzkowski, K. (2015). Real numerical shadow and generalized B–splines, *Lin. Algebra Appl. 479*, 12–51.

Part III

Invited Lecturers

Matrices and the European parliament

Friedrich Pukelshiem

Institute of Mathematics, University of Augsburg, Germany

Abstract

Ever since its inception the European Parliament has been determined to install a unified method for its election. At present, European elections are an aggregate of 28 domestic elections that are conducted within the Union's Member States. However, there is a perfect way to amalgamate past diversity into prospective uniformity: double-proportionality. Double-proportional methods allow to represent two dimensions of the electorate, its partitioning into territorial districts (the Member States) as well as its division by political parties (at European level, which exists on the paper and need to make an appearance in real life). This approach leads us into the happy world of matrices, with Member States as rows and unionwide parties as columns. The lecture will develop theory and practice of translating a matrix of vote counts into a matrix of seat numbers in such a way that the side-conditions that are decreed by the Union's constitutional frame (TEU-Lisbon) are honored.

Keywords: European parliament, elections, double-proportional methods.

References

Pukelsheim, Friedrich (2014), *Proportional Representation - Apportionment Methods and Their Applications*. Springer.

Adjacency preservers

Peter Šemrl

Faculty of Mathematics and Physics, University of Ljubljana, Slovenia

Abstract

Two matrices are said to be adjacent if their difference is of rank one. Fundamental theorems of geometry of matrices describe the general form of bijective maps on various spaces of matrices preserving adjacency in both directions. We will present some recent improvements of these results and discuss connections with geometry and applications in mathematical physics.

Keywords: adjacency, coherency, geometry of matrices, Grassmann space, bounded observable, effect algebra, Minkowski space.

References

Huang, W.-l. and Šemrl, P. (2008). Adjacency preserving maps on hermitian matrices. *Canad. J. Math. 60*, 1050–1066.

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Šemrl, P. (2014). The optimal version of Hua's fundamental theorem of geometry of rectangular matrices. *Mem. Amer. Math. Soc 232*, 74pp.

Part IV

Special session in memoriam of our dearest colleague Glória Cravo

Eigenvalues, multiplicities and graphs: recent advances and open questions

Charles Johnson

College of William and Mary, Williamsburg, USA

Abstract

Let G be an undirected graph on n vertices and let $S(G)$ denote the set of all symmetric matrices with graph G and let $L(G)$ denote the set of all multiplicity lists occurring among the matrices in $S(G)$. The diagonal entries of matrices in $S(G)$ are free. There has long been study of this question that has resulted in certain key theorems and much specific information about lists for certain graphs. The case of trees has remarkable structure and has received considerable attention.

We will review the history of results in the area and then move to some (very) recent advances, dealing with general matrices, over general fields and geometric multiplicities. Then, we will survey some important open questions, and thoughts about them, time permitting.

Part V

Young Scientists Awards of MAT-TRIAD 2013

Computational complexity and interval Linear Algebra

Jaroslav Horáček

Charles University, Faculty of Mathematics and Physics, Department of Applied Mathematics, Prague, Czech Republic

Abstract

More and more scientists are interested in the field called interval analysis. The key idea of this field is replacing numbers with intervals. We might want to do so because of many reasons (verification, taking into account rounding errors). The interesting question is "What happens with linear algebra, if we replace numbers with closed real intervals?" We have to slightly redefine the classical tasks such as checking regularity of a matrix, finding inverse matrix, solving a system of linear equations, deciding whether the same system is solvable, determining spectral radius of a matrix etc.

How does incorporating intervals in our problems change computational complexity? The problems should be of at least the same difficulty as in classical linear algebra, since real numbers are actually intervals with the same lower and upper bound. Unfortunately, solving interval problems often becomes NP-hard. The more it is important to look for special instances of problems, that are easily solvable.

In this talk we explore the classical linear algebraic tasks mentioned earlier and their computational complexity from the perspective of interval linear algebra.

Keywords: interval analysis, interval linear algebra, computational complexity.

References

Hladík, M. (2015). *Complexity issues for the symmetric interval eigenvalue problem.* Open Mathematics.

Kreinovich, V., Rohn, J., Kahl, P., and Lakeyev, A. (1998). *Computational complexity and feasibility of data processing and interval computations.* Springer.

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Rohn, J., and G. Rex. (1996). *Checking properties of interval matrices.* Tech. Rep. 686 of Charles University

H-matrix theory and applications

Ljiljana Cvetković[1] and Maja Nedović[2]

[1]Department of Mathematics and Informatics, Faculty of Science, University of Novi Sad, Serbia
[2]Faculty of Technical Sciences, University of Novi Sad, Serbia

Abstract

The theory of M- and H-matrices has become one of the basic tools in applied linear algebra and it has contributed to different areas of mathematical research and applications. Many results in numerical analysis, eigenvalue localization problems, analysis of iterative methods for solving systems of linear equations came from H-matrix theory. Also, many results in engineering rely on mathematical foundation that is, explicitly or implicitly, formulated in terms of H-matrices.

In this talk, different matrix properties that guarantee nonsingularity of matrices and define different subclasses of H-matrices will be presented together with related results concerning Schur complement matrices, eigenvalue localization and bounds of the max-norm of the inverse matrix.

Keywords: $H-$matrix, eigenvalues, Schur complement, max-norm of the inverse matrix.

References

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Part VI

Invited Mini-Symposia

IMS1

Linearizations and ℓ-ifications of matrix polynomials: theory and applications

Organizers: **Maribel Isabel Bueno (USA)**
Froilán M. Dopico (Spain)
Susana Furtado (Portugal)

Hermitian linearizations of Hermitian matrix polynomials preserving the sign characteristic

Maria I. Bueno[1], Froilán Dopico[2] and Susana Furtado[3,4]

[1]University of California, USA
[2]University Carlos III de Madrid, Spain
[3]University of Porto, Portugal
[4]CEAFEL

Abstract

The most widely used approach to solving the polynomial eigenvalue problem $P(\lambda)x = 0$ is to consider a linearization of the matrix polynomial $P(\lambda)$ and solve the corresponding linear eigenvalue problem. It is important to consider linearizations preserving whatever structure a matrix polynomial $P(\lambda)$ might possess, since such linearizations preserve the properties of the eigenvalues imposed by the structure of $P(\lambda)$.

Hermitian matrix polynomials are one of the most important classes of structured matrix polynomials arising in applications and their real eigenvalues are of great interest.

The sign characteristic of a Hermitian matrix polynomial $P(\lambda)$ with nonsingular leading coefficient is a set of signs attached to the elementary divisors of $P(\lambda)$ associated with the real eigenvalues, which has a key role in determining the behavior of systems described by Hermitian matrix polynomials.

In this talk we present a characterization of all the Hermitian linearizations that preserve the sign characteristic of a given Hermitian matrix polynomial with nonsingular leading coefficient and identify several families of such linearizations that can be easily constructed from the coefficients of the matrix polynomial.

Keywords: eigenvalues, Hermitian matrix polynomial, linearization and sign characteristic.

Structured backward error analysis of polynomial eigenvalue problems solved by linearization

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[2]Department of Computer Science, KU Leuven, Belgium

Abstract

One of the most frequently used techniques to solve polynomial eigenvalue problems is linearization, in which the polynomial eigenvalue problem is turned into an equivalent linear eigenvalue problem with the same eigenvalues, and with easily recoverable eigenvectors. The eigenvalues and eigenvectors of the linearization are usually computed using a backward stable solver such as the QZ algorithm. Such backward stable algorithms ensure that the computed eigenvalues and eigenvectors of the linearization are exactly those of a nearby linear pencil, where the perturbations are bounded in terms of the machine precision and the norms of the matrices defining the linearization. With respect to the linearization, we may have solved a nearby problem, but we would also like to know if our computed solution is the exact solution of a nearby polynomial eigenvalue problem.

We perform a structured backward error analysis of polynomial eigenvalue problems solved via linearization. Through the use of dual minimal bases, we unify the construction of strong linearizations for many different polynomial bases. By inspecting the prototypical linearizations for polynomials expressed in a number of classical bases, we are able to identify a small number of driving factors involved in the growth of the backward error. One of the primary factors is found to be the norm of the block vector of coefficients of the polynomial, which is consistent with the current literature. We derive upper bounds for the backward errors for specific linearizations, and these are shown to be reasonable estimates for the computed backward errors.

Keywords: linearization, backward error, stability, dual minimal bases, strong linearization.

Quasi-Canonical Forms for Matrix Polynomials

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[2]Departamento de Matemáticas, Universidad Carlos III de Madrid, Leganés, Spain
[3]Department of Mathematics, University of Rhode Island, Kingston, Rhode Island, USA
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[5]Department of Mathematical Engineering (INMA/ICTEAM), Université Catholique de Louvain, Louvain-la-Neuve, Belgium

Abstract

The Weierstrass and Kronecker canonical forms for matrix pencils are indispensable tools for obtaining insight into both the theoretical and computational behavior of pencils and their corresponding eigenproblems. The absence of any analogous result for matrix polynomials of higher degree has made it difficult to achieve the same depth of understanding for general matrix polynomials as we have for pencils. In this talk I will describe recent progress towards canonical forms for matrix polynomials, with emphasis on the *quadratic* case. General quadratics (both square and rectangular), as well as various important structure classes such as Hermitian and palindromic quadratic matrix polynomials, will be considered. As time permits, recent progress on regular matrix polynomials of higher degree will also be discussed.

Keywords: matrix polynomials, quasi-canonical forms, inverse problems.

A framework for spectrally equivalent matrix polynomials in non-standard representations

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[2]Department of Mathematics, University of Rhode Island, Kingston, Rhode Island, USA

Abstract

Matrix polynomials $P(\lambda)$ and their associated eigenproblems are fundamental for a variety of applications. Certainly the standard (and apparently most natural) way to express such a polynomial has been

$$P(\lambda) = A_k\lambda^k + A_{k-1}\lambda^{k-1} + \cdots A_1\lambda + A_0$$

where $A_i \in \mathbb{F}^{m\times n}$. However, it is becoming increasingly important to be able to work directly and effectively with polynomials in the non-standard form

$$Q(\lambda) = A_k\phi_k(\lambda) + A_{k-1}\phi_{k-1}(\lambda) + \cdots + A_1\phi_1(\lambda) + A_0\phi_0(\lambda)\,,$$

where $\{\phi_i(\lambda)\}_{i=0}^{k}$ is a non-standard basis for the space of scalar polynomials of degree at most k. This talk will introduce a new framework for systematic construction of families of strong linearizations for matrix polynomials like $Q(\lambda)$, with emphasis on the classical bases associated with the names Newton, Bernstein, and Lagrange. Time permitting, we will show that this framework also enables us to construct families of companion ℓ-ifications for matrix polynomials expressed in the standard basis and whose grade is a positive multiple of ℓ.

Keywords: matrix polynomials, non-standard representations, strong linearizations.

On the inverse eigenvalue problem for $T-$alternating and $T-$palindromic matrix polynomials

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Abstract

The inverse polynomial eigenvalue problem for regular matrix polynomials can be expressed as follows: Given a list of finite and infinite elementary divisors, construct a matrix polynomial hat has exactly this list of elementary divisors. In the talk, we consider this problem for matrix polynomials with $T-$alternating or $T-$palindromic structure. In the case that no infinite elementary divisors are prescribed, we give necessary and sufficient conditions on lists of elementary divisors so that there exist a $T-$alternationg resp. $T-$palindromic matrix polynomial with the given list as its elementary divisors. Moreover, we show that the matrix polynomial can be chosen to be in a specific form, the so-called anti-triangular form.

Keywords: inverse eigenvalue problem, alternating matrix polynomials, Palindromic matrix polynomials.

Generalization of Newton linearization for polynomial equations described by Birkhoff data

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Abstract

In scientific computation, we are often interested in the roots of a polynomial that is not presented in the monomial basis. One well-known example of this occurs in the context of interpolating polynomials when the derivatives of the polynomial are available. Here the value of a function is considered as the zeroth order derivative. If the orders of the derivatives form an unbroken sequence, we deal with Hermite interpolation, and if some of the sequences are broken, Birkhoff interpolation comes into play.

In this research, we concentrate on polynomials specified by Birkhoff interpolation data. The customary formulation of this problem has been done through a so-called incidence matrix. In this contribution a Birkhoff matrix is introduced. The characterization of the interpolation problem in terms of this Birkhoff matrix is presented. In particular, in the presence of confluent nodes, the structure of the polynomial as well as companion matrices is as straightforward as for distinct nodes.

Keywords: linearization, polynomial root-finding, matrix pencils, Birkhoff interpolation.

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Constructing strong ℓ-ifications

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Abstract

We present in this talk some recent development in the theory of strong ℓ-ifications of matrix polynomials. More precisely, we present an algorithm to construct strong ℓ-ifications of a given matrix polynomial $P(\lambda)$ of degree d and size $m \times n$ using only the coefficients of the polynomial and the solution of linear systems of equations. Our construction is valid for a wider situation than the ones considered previously in the literature, which were limited to the case where ℓ divides d. In particular, the construction is valid for the case where ℓ divides one of nd or md. In the case where ℓ divides nd (respectively, md), the strong ℓ-ifications we construct allow us to easily recover the minimal indices of $P(\lambda)$. In particular, we show that they preserve the left (resp., right) minimal indices of $P(\lambda)$, and the right (resp., left) minimal indices of the ℓ-ification are the ones of $P(\lambda)$ increased by $d - \ell$ (each). Moreover, in the particular case where ℓ divides d, the new method provides a companion ℓ-ification that resembles very much the companion ℓ-ifications already known in the literature.

Keywords: matrix polynomials, minimal indices, linearizations, ℓ-ifications, dual minimal bases.

References

Terán, F. De , Dopico, F. M. and Van Dooren, Paul M., *Constructing strong ℓ-ifications from dual minimal bases*. Submitted.

Linearizations of rational function matrices

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Abstract

Given a rational matrix $R(\lambda)$ (i.e., a matrix whose elements are quotients of coprime polynomials with real or complex coefficients), the rational eigenvalue problem (REP) is to find scalars λ and nonzero vectors x satisfying

$$R(\lambda)x = 0.$$

The scalars λ and the vectors x are called, respectively, eigenvalues and eigenvectors of the rational matrix $R(\lambda)$.

The REP arises in many applications and several approaches can be used to tackle it. One of them is to reduce the REP to a polynomial eigenvalue problem (PEP) by multiplying $R(\lambda)$ by the least common multiple of the denominators, $d(\lambda)$ say, and then into a linear eigenvalue problem by linearizing the obtained PEP. This may be a practical approach if the degree of $d(\lambda)$ is small. Another possibility is to use a nonlinear eigensolver (like Newton's method, nonlinear Rayleigh quotient method, nonlinear Jacobi-Davidson method or nonlinear Arnoldi method) to solve the REP directly.

In a recent paper [Su and Bai, (2011)] a new method is proposed that, using very basic realization theory for linear control systems, yields a linearization out of the given $R(\lambda)$ that preserves the finite zeros of $R(\lambda)$. The goal of this talk is to provide a theoretical framework for Su and Bai's approach. Specifically, a definition of linearization of a rational function matrix will be proposed that naturally extents the well-known one for polynomial matrices and that preserves both the finite and infinite structure. Then some basic properties of linearizations will be presented.

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IMS2

Spectral graph theory

Organizer: Domingos Moreira Cardoso (Portugal)

Ky Fan theorem applied to Randić energy

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Abstract

Let G be an undirected simple graph of order n with vertex set $V = \{v_1, \ldots, v_n\}$. Let d_i be the degree of the vertex v_i . The Randić matrix $\mathbf{R} = (r_{i,j})$ of G is the square matrix of order n whose (i, j)-entry is equal to $1/\sqrt{d_i\, d_j}$ if the vertices v_i and v_j are adjacent, and zero otherwise. The Randić energy is the sum of the absolute values of the eigenvalues of $\mathbf{R}$. Let $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$ be matrices, such that $\mathbf{X} + \mathbf{Y} = \mathbf{Z}$. Ky Fan established an inequality between the sum of singular values of $\mathbf{X}$, $\mathbf{Y}$, and $\mathbf{Z}$. We apply this inequality to obtain bounds on Randić energy. Some results are presented considering the energy of a symmetric partitioned matrix, as well as an application to the coalescence of graphs.

Keywords: graph spectra, graph energy.

References

Gutman, I., Martins, E. A., Robbiano, M. and Martín, B. S. (2014). Ky Fan Theorem Applied to Randić Energy. *Linear Algebra Appl. 459*, 23–42.

Determination of regular exceptional graphs by $(\kappa, \tau)-$extensions

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Abstract

An exceptional graph is a connected graph with least eigenvalue not less than -2 which is not a generalized line graph. A $(\kappa, \tau)-$regular set is a vertex subset S inducing a $\kappa-$regular subgraph such that every vertex not in S has τ neighbors in it. We present a new construction technique that introduces $(\kappa, \tau)-$regular sets in regular graphs, called $(\kappa, \tau)-$extension, which induces a partial order between them. It is shown that the process of extending a graph is reduced to the construction of the incidence matrix of a combinatorial 1− design, considering several rules to prevent the production of isomorphic graphs. The construction of the set of regular exceptional graphs, which are partitioned in three layers, is the recursive application of such $(\kappa, \tau)-$extensions. We conclude that the independence number attains Hoffman's upper bound for the graphs of 1st and 2nd layers and the set of regular exceptional graphs has a partially ordered set structure whose Hasse diagram is presented.

Keywords: spectral graph theory, exceptional graphs, posets.

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Limit points for the second largest eigenvalues of $H_{p,q}$ graphs

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Abstract

The study of limit points for the eigenvalues of graphs was iniciated by Hoffman, in 1972. Later, Guo and Kirkland also studied of limit points for some Laplacian eigenvalues of graphs.

For integer parameters $1 \leq p \leq q$ we define a graph $H_{p,q}$ resultant from an operation applied to two copies of K_p and a copy of $K_{q,q}$. For fixed p, we characterize the limit point for the second largest eigenvalue of graphs $H_{p,q}$. For each p, it is equal to $2p-1$.

Keywords: graphs, adjacency matrix, second largest eigenvalue, limit point.

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Adjacency and Laplacian spectra of powers of lexicographic products of graphs

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Abstract

Arbitrary iterations of the lexicographic product of graphs are considered and their adjacency (Laplacian) spectra of regular (arbitrary) graphs is characterized. Namely, the adjacency (Laplacian) spectra of powers, that is, iterated products, of regular (arbitrary) graphs is determined. These characterizations are based on the results published by Cardoso, Freitas, Martins and Robbiano 2013. Several properties and graph invariants of these products are also presented and a few related applications in chemistry and nanomaterials are explored.

Keywords: graph spectra, graph operations.

References

Cardoso, D. M., Freitas, M. A. de, Martins, E. A. and Robbiano, M. (2013). Spectra of graphs obtained by a generalization of the join graph operation. *Discrete Mathematics 313*, 733–741.

On trees with non-main least eigenvalue

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Abstract

An eigenvalue of the adjacency matrix of a graph G (herein called an *eigenvalue of* G) is said to be *main* if the all-1 vector is not orthogonal to the associated eigenspace. Otherwise it is called *non-main*. In this work, we determine the main eigenvalues of a path on n vertices and show that its least eigenvalue is non-main if and only if n is even. We identify the main eigenvalues of diameter three trees and show that, for these trees, there exists a relation between the number of main eigenvalues and the fact of the least eigenvalue to be or not a main eigenvalue. Assuming that λ is the least eigenvalue of a graph G, we also investigate when $-1-\lambda$ is the largest or second largest eigenvalue of the complement of G.

Keywords: main/non-main eigenvalues, spectrum of a tree.

On a conjecture for the distance Laplacian matrix

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Abstract

In this work we determine all connected graphs of order n that have some distance Laplacian eigenvalue with multiplicity $n-2$. In particular, we prove that if the largest eigenvalue of the distance Laplacian matrix of a connected graph G of order n has multiplicity $n-2$, then $G \cong S_n$ or $G \cong K_{p,p}$, where $n = 2p$. This result solves a conjecture proposed by M. Aouchiche and P. Hansen in "A Laplacian for the distance matrix of a graph, in Czechoslovak Mathematical Journal, 64(3): 751-761, 2014". Moreover, we prove that if G has P_5 as an induced subgraph then the multiplicity of the largest eigenvalue of the distance Laplacian matrix of G is less than $n-3$.

Keywords: distance Laplacian matrix, Laplacian matrix, largest eigenvalue, multiplicity of eigenvalues.

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Distance Laplacian and distance signless Laplacian integral graphs

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Abstract

For a connected graph G, we denote by $D(G)$ the distance matrix of G and, by $T(G)$, the transmission matrix of G, the diagonal matrix of the row sums of $D(G)$. The matrices $D^L(G) = T(G) - D(G)$ and $D^Q(G) = T(G) + D(G)$ are called the distance Laplacian and the distance signless Laplacian of G, respectively. One important problem investigated in Spectral Graph Theory is to characterize graphs for which all eigenvalues of M are integers, where M is a matrix associated to the graph. In this work we discuss M-integrality for some special classes of graphs, where $M = D^L(G)$ or $M = D^Q(G)$. In particular, we consider the classes of complete split graphs, multiple complete split-like graphs, extended complete split-like graphs and multiple extended complete split-like graphs, giving conditions for the $M-$ integrality in each of them.

Keywords: distance Laplacian Matrix, distance signless Laplacian matrix, $M-$integral graph.

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Randić spectra of the $H-$join graph and Randić energy of graphs with clusters

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Abstract

Let H be an undirected simple graph with vertices $v_1, \ldots, v_k$ and let $G_1, \ldots, G_k$ be a sequence formed with k disjoint graphs G_i, $1 \leq i \leq k$. The $H-$generalized composition (or $H-$join) of this sequence, denoted by $H[G_1, \ldots, G_k]$, is a graph such that every vertex of G_i is adjacent to every vertex of G_j whenever the vertex v_i is adjacent to vertex v_j in H. A characterization of the spectrum and the Laplacian spectrum of $H[G_1, \ldots, G_k]$ was given by Cardoso, Freitas, Martins and Robbiano 2013. In this talk we extend the results obtained by those authors to the characterization of the Randić and the Normalized Laplacian spectrum of $H[G_1, \ldots, G_k]$. As an application, considering a graph G with a $b-$cluster of order k (that is, an independent set of cardinality k such that every two vertices share the same b neighbours), some bounds on the Randić energy of G are presented.

Keywords: graph eigenvalues, Randić matrix, normalized Laplacian matrix, Randić energy, clusters.

References

Cardoso, D. M., Freitas, M. A. de, Martins, E. A. and Robbiano, M. (2013). Spectra of graphs obtained by a generalization of the join graph operation. *Discrete Mathematics 313*, 733–741.

Integral graphs with at most two vertices of degree larger than 2

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Abstract

The spectrum of a graph G is the spectrum of its adjacency matrix. A graph G is called integral when its spectrum is integral, i.e., all eigenvalues of the adjacency matrix of G are integer values. In this talk, we present all integral graphs when G has at most two vertices of degree larger than 2.

Keywords: spectral graph theory, adjacency matrix, spectrum, integral graphs.

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Importance of a special class of Jordan subspaces in the synchrony phenomenon of networks

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Abstract

Networks are studied in many different areas of science and examples are numerous and varied. From the dynamical point of view in networks, it is of interest to study when distinct individuals exhibit identical dynamics, at all instants of time, being synchronized. For example, in the 17th century, the physicist Christiaan Huygens, inventor of the pendulum clock, was surprised by the synchronization of the motion of two pendula. In nature, one of the most spectacular examples occurs when thousands of male fireflies gather in trees at night and flash in unison to attract females, providing a silent, hypnotic concert.

Dynamical systems (systems of ordinary differential equations) that are consistent with the structure of a given network are called the coupled cell systems. The existence of certain flow-invariant subspaces, defined in terms of equalities of certain cell coordinates—the *synchrony subspaces*—, for all the associated coupled cell systems, is forced by the network.

Given a regular network (digraph with only one type of nodes, one type of edges and with all nodes receiving the same number of edges), we single out a special class of Jordan subspaces [Gohberg et al., 2006] of the corresponding adjacency matrix and we use it to study the synchrony phenomenon in networks. To be more precise, we prove that a subspace defined by the equality of some cell coordinates is a synchrony subspace of a regular network if and only if it is a direct sum of subspaces in that special class.

Keywords: networks, Jordan subspaces, synchrony.

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The $k-$regular induced subgraph problem for $k = 1, 2$

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Abstract

The problem of finding a maximum k-Regular Induced Subgraph (k-RIS) is a NP-hard graph problem. For some values of k we obtain some well known problems. For $k = 0$ finding the maximum 0-RIS of a graph G corresponds to find the independence number of G, $\alpha(G)$. For $k = 1$ a 1-RIS is known as strong-matching or induced matching and for $k = 2$ the 2-RIS problem is equivalent to determine cycles of shortest length herein defined as induced cycle cover. In this talk we present an integer programming formulation for this problem. For 1-RIS we introduce a combinatorial upper bound and prove that it is attained when the graph is a tree. We also prove the integrality of some particular polytopes when $k = 1$ and $k = 2$. A computational study is presented comparing some known bounds with bounds obtained from the integer programming relaxations.

Keywords: integer programming, strong-matchings, induced cycles, combinatorial optimization.

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Effects on some graph invariants by adding edges among the vertices of clusters

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Abstract

Let G be a simple undirected graph. A cluster $C(s,l)$ of G is an independent set $s \geq 2$ vertices of G having the same set of neighbors with cardinality l. Let G be a connected graph which includes a cluster $C(s,l)$. Let H be a graph of order s. Let $G(H)$ be the graph obtained from G and H identifying the vertices of H with the vertices of $C(s,l)$. Let $M(R)$ be the Laplacian matrix or the signless Laplacian matrix or the adjacency matrix of a graph R. If the all $1-$ vector of order s is an eigenvector of $M(H)$, it is shown that $M(G(H))$ is orthogonally similar to a 2×2 block diagonal matrix in which one of the blocks is a diagonal matrix. This result is used to study the effects on the algebraic connectivity, Laplacian index, Kirchhoff index, adjacency index, energy and other graph invariants, by adding edges among the vertices of each cluster of a given graph.

Keywords: algebraic connectivity, Laplacian index, Kirchhoff index, adjacency index, energy.

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In search of graphs with uniform rank spread two

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Abstract

For a simple graph $G = (\mathcal{V}, \mathcal{E})$ with vertex-set $\mathcal{V} = \{1, \ldots, n\}$, let $\mathscr{S}(G)$ be the set of all real symmetric n-by-n matrices A whose graph is G. For a graph with minimum rank $\mathrm{mr}(G)$, the maximum multiplicity $M(G)$ of eigenvalues over $\mathscr{S}(G)$ is $n - \mathrm{mr}(G)$. Relative to A and eigenvalue λ, a λ–core vertex i of G has a non–zero entry at i on some λ–eigenvector, while a λ–core–forbidden vertex ℓ of G has a zero entry at ℓ on all λ–eigenvectors. A λ–core–graph has no λ–core–forbidden vertices. Spectral graph theory elucidates the effect on $M(G)$ when deleting core or core–forbidden vertices. The rank spread $r_v(G)$ of G at a vertex v, defined as $\mathrm{mr}(G) - \mathrm{mr}(G - v)$, can take values $\varepsilon \in \{0, 1, 2\}$. In general, distinct vertices in a graph may assume any of the three values. For $\varepsilon = 0$ or 1, there exist graphs with uniform $r_v(G)$ (equal to the same integer at each vertex v). We show that only for $\varepsilon = 0$, will a single matrix $\mathbf{A}$ in $\mathscr{S}(G)$ determine when a graph has uniform rank spread. Moreover, a graph G, with vertices of rank spread zero or one only, is a λ–core graph for a λ–optimal matrix $\mathbf{A}$ (having rank $\mathrm{mr}(G)$) in $\mathscr{S}(G)$. We seek to answer the open question: *Do graphs of uniform rank spread two exist?*

Keywords: minimum rank of graphs, maximum eigenvalue multiplicity, core vertices, rank spread.

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The spectrum of an $I-$graph

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Abstract

For integers $n \geq 3$ and $1 \leq j, k < \frac{n}{2}$, the I-graph $I(n, j, k)$ is the graph with vertex set $\{a_0, a_1, \ldots, a_{n-1}, b_0, b_1, \ldots, b_{n-1}\}$ and edge set

$$\{\{a_i, a_{i+j}\}, \{a_i, b_i\}, \{b_i, b_{i+k}\} : i = 0, \ldots, n-1\},$$

with subscripts reduced modulo n. The class of I-graphs arised as a natural generalization of the so called [Coxeter, 1950] *generalized Petersen graphs* and has attracted the attention of many graph theorists. Considerable study of I-graphs, under algebraical, combinatorial and geometric approaches can be found, among others, in the articles [Bobben et al., 2005], [Bonvicini and Pisanski, 2013], [Frucht et al., 1971], [Horvat et al., 2012], [Petkovšek and Zakrajšek, 2009] and references therein. In our work we investigate the I-graphs under an spectral approach: we completely determine the spectrum of an I-graph, that is, the eigenvalues of its adjacency matrix, by using properties of circulant block matrices. Furthermore, we apply our result and spectral graph techniques to give new proofs of necessary and sufficient conditions for bipartiteness and connectedness of arbitrary I-graphs. Also, we establish the nullity, that is, the dimension of the eigenspace associated to null eigenvalue, of certain I-graphs.

Keywords: I-graph, generalized Petersen graph, adjacency matrix of a graph.

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IMS3

Algebraic methods in operator theory

Organizer: M. Cristina Câmara (Portugal)

From Jacobson's lemma to common spectral properties of operators

Chafiq Benhida

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Abstract

Starting from an algebraic result in a ring E with identity 1, known as Jacobson's lemma that's the equivalence: $1 - ab$ is invertible if and only if $1 - ba$ invertible, we 'll present its applications in spectral theory for bounded linear operators on Banach spaces and also its extensions and generalizations for operators and multioperators.

Keywords: n-tuples, Taylor spectrum, Joint spectra, $SVEP$, Bishop's property (β).

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Non-self-adjoint graphs

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Abstract

On finite metric graphs we consider Laplace operators, subject to various classes of non-self-adjoint boundary conditions imposed at graph vertices. We investigate spectral properties, existence of a Riesz basis of projectors and similarity transforms to self-adjoint Laplacians. Among other things, we describe a simple way to relate the similarity transforms between Laplacians on certain graphs with elementary similarity transforms between matrices defining the boundary conditions.

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Factorization of matrices in $Q-$ classes

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Abstract

We study classes of invertible and essentially bounded 2×2 matrix functions G in the real line satisfying

$$G^T Q_1 G = \det G \cdot Q_2, \tag{1}$$

where Q_1 and Q_2 are symmetric matrix functions, meromorphic in the lower and upper half-plane, respectively. We take advantage of the algebraic nature of their definition and properties to obtain several equivalence representations for each class and explore their close connection with certain non-linear scalar equations. The results are applied to study various factorization problems.

This is based on a joint work with M. Cristina Câmara.

Operator theory on C^*-algebras via local multipliers

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Abstract

The local multiplier algebra of a C^*-algebra is a noncommutative infinite-dimensional analogue of the field of fractions of an integral domain. Together with Pere Ara (Barcelona) we developed a rather extensive theory of local multipliers and laid down the foundations in our monograph in Springer-Verlag 2003. I shall report on some recent applications of this tool to the study of linear operators between C^*-algebras.

Functoriality for the reduced C^*-algebra of $GL(n, \mathbb{R})$

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Abstract

The local Langlands correspondence for reductive groups over archimedean fields dates back to 1973 although it was published later in 1989. In Langlands conjectures, functoriality is a far-reaching principle, with implications in representation theory, number theory and beyond. In this talk we investigate functoriality for the reduced C^*-algebra of $GL(n, \mathbb{R})$ at the level of $K-$theory, uncovering a relationship between the Baum-Connes correspondence and the principle of functoriality. Joint work with Roger Plymen.

Keywords: reduced C^*-algebra, Langlands functoriality, representation ring.

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Reflexive spaces of operators

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Abstract

It is well known that the weakly closed bimodules of certain reflexive operator algebras can be completely characterised in terms of order homomorphisms of the lattice of invariant subspaces of the algebra. This talk addresses the possibility of extending this type of characterisation to any reflexive space $\mathcal{M}$ of operators on Hilbert space. It will be shown that to succeed along this lines it suffices to go from order homomorphisms of lattices to order homomorphisms of bilattices of subspaces that are determined by $\mathcal{M}$ in an appropriate manner.

This is a joint work with J. Bračič (University of Ljubljana).

Keywords: reflexive space of operators, order-preserving map.

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On the hyper– and 2– hyperreflexivity of power partial isometries

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Abstract

A power partial isometry is an operator for which all its powers are partial isometries. There are a lot of examples of such operators: a single Jordan block, sums (finite or not) of Jordan blocks, unilateral or backward shifts. The concept of reflexivity and hyperreflexivity arises from the problem of existence of a nontrivial invariant subspace for an operator on a Hilbert space. An operator is called reflexive if it has so many invariant subspaces that they determine the membership in the algebra generated by the given operator. An operator is hyperreflexive (much stronger property than reflexivity) if the usual distance from any operator to the algebra generated by the given operator can be controlled by the distance given by invariant subspaces; in other words by distance given by rank 1 operators in the preanihilator. Replacing rank one operators by rank two the 2– hyperreflexivity can be defined.

The 2– hyperreflexivity of power partial isometries are shown. Necessary and sufficient conditions for hyperreflexivity of completely non–unitary power partial isometries are given.

Joint work with K. Piwowarczyk.

Keywords: reflexivity, Jordan blocks, invariant subspace.

Matrix Fourier symbols in problems of wave diffraction by a strip with higher order impedance conditions

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Abstract

The main purpose of this work is to analyse an impedance boundary-transmission problem for the Helmholtz equation originated by a problem of wave diffraction by an infinite strip with higher order imperfect boundary conditions. A constructive approach of matrix operator relations is built, which allows a transparent interpretation of the problem in an operator theory framework. In particular, different types of matrix operator relations are exhibited for different types of operators acting between Lebesgue and Sobolev spaces on a finite interval and on the positive half-line. At the end, we describe when the operators associated with the problem enjoy the Fredholm property with Fredholm index equal to zero in terms of the initial space order parameters.

This is a joint work with L. P. Castro.

This work was partially supported by the Center of Mathematics and Applications of University of Beira Interior (CMA-UBI) through the project UID/MAT/00212/2013.

Keywords: boundary value problem, Helmholtz equation, Bessel potential space, convolution type operator, Fredholm operator, higher order impedance boundary condition, wave diffraction, Wiener-Hopf operator.

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Numerical range of linear pencils with one Hermitian coefficient

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Abstract

Let A,B be $n \times n$ (complex) matrices. Recall that the numerical range of a linear pencil of a pair (A, B) is the set

$$W(A, B) = \{x^*(A - \lambda B)x : x \in \mathbb{C}^n, \|x\| = 1, \lambda \in \mathbb{C}\}.$$

The numerical range of linear pencils with hermitian coefficients was studied by some authors.

We are mainly interested in the study of the numerical range of a linear pencil, $A - \lambda B$, when one of the matrices A or B is Hermitian and $\lambda \in \mathbb{C}$. We characterize it for small dimensions in terms of certain algebraic curves. For the case $n = 2$, the boundary generating curves are conics. For the case $n = 3$, all the possible boundary generating curves can be completely described by using Newton's classification of cubic curves. The results are illustrated by numerical examples.

Keywords: numerical range, linear pencil, generalized eigenvalue problem, plane algebraic curve.

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Structured pseudospectra of block matrices

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Abstract

The ϵ-pseudospectrum of a matrix A consists of all eigenvalues of matrices whose distance from A is less than ϵ. If the given matrix A is of certain structure, it is also natural to consider only perturbations of the same structure, which gives rise to the notion of the structured pseudospectrum of A that was first introduced by Böttcher, Grudsky, and Kozak. In the second half of the 2000s, Graillat and Rump showed that the unstructured and structured pseudospectra coincide for many structures, such as Toeplitz, Hankel, and symmetric, while examples of structures for which they differ include Hermitian and skew-Hermitian structures.

In this talk we consider the question of determining block structures for which the result of Graillat and Rump remains true, and illustrate our study with numerical computations using EigTool and SeigTool. This talk is based on joint work with Richard Ferro.

IMS4

Coding theory

Organizers: Raquel Pinto (Portugal)
Diego Napp (Portugal)

Superregular matrices with applications to convolutional codes

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Abstract

We say that a matriz is superregular if all its minors that are not trivially zero are nonzero. This definition generalizes previous notions of superregularity applied to full matrices or lower triangular matrices whose entries are in a finite field, used in the context of coding theory to generate codes with good distance proprieties. The main result of this talk states that such superregular matrices have the property that any linear combination of their columns have the maximum number of nonzero entries that is possible for the given configuration of zeros. A set of sufficient conditions for a matrix to be superregular is given, and these conditions allow us to construct convolutional codes of any rate and Forney indices $\nu_1, \ldots, \nu_k$ with the maximum possible distance.

Keywords: superregular matrix, convolutional codes, Forney indices, optimal distance.

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On MDR codes over a finite ring

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Abstract

In this work, we study codes over the ring $\mathbb{Z}_{p^r}$ satisfying two distinct Singleton type bounds. We characterize a Maximum Distance with respect to the Rank (MDR) code by the smallest free code containing it. This characterization generalizes the one given in 2 for free MDR codes.

Keywords: MDR code, singleton type bounds.

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The fast correlation attack to linear feedback shift registers as a syndrome decoding problem via representation technique

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Abstract

A natural way of analyzing the stream of bits $\boldsymbol{y} = (y_0, y_1, \ldots, y_N)$ produced by a Linear Feedback Shift Register (LFSR) is to understand it as an autonomous system

$$\boldsymbol{x}_{i+1} = A\boldsymbol{x}_i, \ \boldsymbol{y}_i = C\boldsymbol{x}_i, \quad \text{for } i = 0, 1, 2, \ldots \tag{1}$$

for certain binary matrices A and C and inner states $\boldsymbol{x}_i$.

One of the most successful attacks against a secret random sequence of bits produced by certain LFSRs has been achieved by the Fast Correlation Attack by Meier and Staffelbach.

The correlation attack is often viewed as a decoding problem. Assume that a sequence $\boldsymbol{y}$ produced by a LFSR is sent through a transmission channel. Let $\boldsymbol{z}$ be the received channel output, which is correlated to the sequence $\boldsymbol{y}$ with correlation probability p. The sequence $\boldsymbol{y}$ can be interpreted as a codeword in the binary $[N, l]$-code $\mathcal{C}$ generated by the observability matrix G of the autonomous system (1) and the keystream sequence $\boldsymbol{z}$. The problem of the attacker can be formulated as follows: Given a received word $\boldsymbol{z}$, find the transmitted codeword $\boldsymbol{y}$.

Taking advantage of these approach, we analyze the Fast Correlation Attack as a syndrome decoding problem via the representation technique of the syndromes (see Becker et al.).

Keywords: stream ciphers, linear feedback shift registers, cryptanalysis, syndrome decoding problem.

References

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Column distances for convolutional codes over finite rings

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Abstract

In this paper we study column distances of convolutional codes over finite rings. Maximal possible growth in the column distances means that these codes have the potential to have a maximal number of errors corrected per time interval which make them very appealing for sequential decoding, e.g., in multimedia streaming. These results extend previous results on columns distances of convolutional codes over finite fields.

Keywords: convolutional codes, finite rings, column distances.

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IMS5

Matrix theory, applications and engineering

Organizer: Marko Stošić (Portugal)

On some matrix optimization problems arising in computer vision

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[3]Wrocław School of Information Technology, Poland
[4] ISR – Lisbon, Portugal

Abstract

In this talk we consider three different optimization problems with applications in computer vision: (i) the sub-Stiefel Procrustes problem; (ii) the Plücker correction problem; and (iii) the problem of finding the closest essential generalized matrix. With respect to the problem (i), we present an iterative method for computing its solution and investigate the properties of sub-Stiefel matrices. For problem (ii), we give an explicit solution and show that this new approach performs considerably better than the previous ones. About the problem (iii), some strategies to find its solution are discussed.

Keywords: sub-Stiefel matrices, sub-Stiefel Procrustes problem, Plücker coordinates, generalized essential matrices.

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Low rank approximations in computer vision

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Abstract

The use of rank criteria has been pervasive in computer vision applications. Problems such as 3D scene reconstruction from a sequence of (discrete) images or recovering the surface of an object from images with varying illumination can be well approximated recurring to bilinear (observation) models. Occlusions and errors correspond to missing entries which must be estimated. So, in essence, these processes can be formalized as matrix factorization problems of partially prescribed matrices constrained to special manifolds (e.g. stiefel). In this talk we will introduce several computer vision problems that pose challenges both from the mathematical modelling and computational point of view. Of particular interest we highlight the so-called "correspondence problem" (matching points in different images) involving a combinatorial problems in "low rank" models.

Equiangular tight frames, and beyond

Marko Stošić

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Abstract

The problem of equiangular tight frames (ETFs) consists of finding N lines in d-dimensional spaces such that the angle between any two distinct lines is as large as possible. This problem has many different applications, including antenna communications, signal processing and quantum cryptography. Different particular cases turn out to be related to numerous different mathematical techniques and topics. We will review some of the aspects of this problem and recent developments, in particular related to maximal ETFs. Finally, we will present open problems related to ETFs as well as their higher-dimensional extensions.

Distributed processing for multi-agent systems

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Abstract

We overview recent developments in the design and analysis of multi-agent systems that build on the interplay between linear algebra, optimization and probability theory. We illustrate the applications of these mathematical tools in the context of some engineering applications from machine learning, distributed signal processing and control. We point out several open problems that could be of some interest for applied mathematicians.

On classical modal control of quadratic systems

Ion Zaballa

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Abstract

Classical modal control refers to the possibility of driving a quadratic system to quadratic diagonal form by strict equivalence. This amounts to finding non-singular square matrices P and Q such that $PL(\lambda) = \widetilde{L}(\lambda)Q$ where $L(\lambda) = M\lambda^2 + D\lambda + K$ is the given system and $\widetilde{L}(\lambda) = \widetilde{M}\lambda^2 + \widetilde{D}\lambda + \widetilde{K}$ is a diagonal quadratic matrix polynomial.

The classical result by Caughey and O'Kelly (1965) gives a necessary and sufficient condition when $L(\lambda)$ is symmetric and M is positive definite. Ma and Caughey (1995) studied this problem for general systems and Lancaster and Zaballa (2009) provided a solution for symmetric systems when the pencil $\lambda M + K$ is semisimple and its eigenvalues are of definite type, and for general systems when $\lambda M + K$ has simple eigenvalues. In all these cases, the necessary and sufficient condition for reducing a given system to diagonal form by strict equivalence has the following commutative expression: $KM^{-1}D = DM^{-1}K$.

Recently, the notion of Filters connecting two isospectral quadratic systems has been developed [Garvey et al., (2013) and preprint]. Based on this concept new and more general necessary and sufficient conditions in terms of the spectral data of the given system can be provided. The aim of this talk is to present these new conditions.

This talk is based on joint work with S. Garvey, P. Lancaster, A. Popov and U. Prells.

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IMS6

Functions of matrices

Organizers: Pedro Freitas (Portugal)
Sónia Carvalho (Portugal)

Corona problems, matrix factorization and truncated Toeplitz operators

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Abstract

We explore the connections between the corona theorem, Wiener-Hopf factorization, and Toeplitz operators with matrix symbols, to study spectral properties of truncated Toeplitz operators.

This talk is based on joint work with Jonathan Partington.

Keywords: truncated Toeplitz operator, Toeplitz operator, equivalence after extension, matrix factorization, Corona theorem.

On the norm of the derivatives of symmetric tensor powers

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[2]CEAFEL, Universidade de Lisboa

Abstract

An upper for the norm for the higher order derivatives of the ξ-symmetric tensor power In recent papers we have already obtained formulas for directional derivatives, of all orders, of the immanant and of the m-th ξ-symmetric tensor power of an operator and a matrix, when ξ is an irreducible character of the full symmetric group. The operator bound norm of these derivatives was also calculated. In this talk it will be presented other results that have been established for every symmetric tensor power associated with a character of a subgroup.

Keywords: norm of a multilinear operator, multilinearity partition, derivative, ξ-symmetric tensor power.

Supercaracter theories for algebra groups defined by involutions

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Abstract

The notion of supercaracter of an algebra group originated with the work of C. André and was then axiomatized by Diaconis and Isaacs. In his doctoral thesis, A. Neto established supercharacter theories for the orthogonal and the symplectic groups (subgroups of the unitriangular group with entries in a finite field). In this seminar we present a generalization of these theories for algebra groups defined by involutions and give examples in matrix groups.

Keywords: supercharacters and superclasses, character theory, nilpotent matrix groups.

More on the Hankel Pencil Conjecture

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Abstract

With x an indeterminate, and $c_i \in \mathbb{C}^*$, define the $n \times n$ Hankel matrix

$$H_n(x) = \begin{bmatrix} & & & x & c_1 & c_2 \\ & & x & c_1 & c_2 & c_3 \\ & & & & \vdots & \vdots \\ x & c_1 & \cdots & \cdots & c_{n-2} & c_{n-1} \\ c_1 & c_2 & \cdots & \cdots & c_{n-1} & c_n \\ c_2 & c_3 & \cdots & \cdots & c_n & c_{n+1} \end{bmatrix}.$$

Conjecture (Hankel Pencil Conjecture HPnC). If $\det H_n(x) \equiv 0$, then the last two columns are dependent.

As Schmale and Sharma [Schmale and Sharma, 2004] showed, truth of this conjecture would imply further confirmation of the 1981 conjecture of Bumby, Sontag, Sussman, and Vasconcelos in Control Theory according to which $\mathbb{C}[y]$ is a feedback cyclization ring. In [Kovačec and Gouveia, 2009], the conjecture was shown to be implied by another conjecture which was called there Rootconjecture RnC and which was shown to be in principle decidable for every individual n via the solution of a particular set of polynomial equations. Although we could prove this way R4C,...,R8C, and hence HP4C,...,HP8C via Groebner basis computations, a major obstacle was that the equations had to be computed for each n. They showed - in dependence of n - no particular pattern. Recently we found - for now conjecturally - equivalent systems of equations which are parametrized by n. These are reasonably elegant, quasi-homogeneous systems which promise progress.

Independent of matrix theory, the Rootconjecture can also be formulated as a conjecture on polynomials that have a simple inductive definition. It is due to Miguel M. R. Moreira, medalist in various recent International Matematical Olympiads.

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On the inverse field of values problem

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Abstract

The field of values of a linear operator is the convex set in the complex plane comprising all Rayleigh quotients. For a given complex matrix, Uhlig proposed the inverse field of values problem: given a point inside the field of values determine a unit vector for which this point is the corresponding Rayleigh quotient. In the present note we propose an alternative method of solution to those that have appeared in the literature. Our approach builds on the fact that the field of values can be seen as a union of ellipses under a compression to the bidimensional case, in which case the problem has an exact solution.

Keywords: field of values, inverse problem, generating vector, compression.

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IMS7

Linear preserver problems

Organizers: Henrique F. da Cruz (Portugal)
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Commutativity preserving maps

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Abstract

Maps on different spaces which preserve commutativity in both directions and have some additional properties, for example linearity, were studied by many authors. Recently some authors were able to characterize maps that preserve commutativity in both directions without a linearity assumption and our aim is to show even more general result.

We will present the characterization of surjective maps on $n \times n$ matrices over algebraically closed fields which preserve commutativity relation only in one direction and have no additional structure like additivity.

We obtained our result using techniques which combine graph theory, linear algebra, and projective geometry.

Keywords: matrix algebra, centralizers, commuting graph, homomorphisms of commuting graphs.

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Dolinar, G. and Kuzma, B., Homomorphisms of commutativity relation. *Linear and Multilinear Algebra.* Accepted for publication.

Immanants of doubly stochastic matrices

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Abstract

Let Ω_n denote the set of the doubly stochastic matrices, that is, the set of the $n \times n$ matrices S with nonnegative real entries and all row and column sums equal to 1. It is well known that Ω_n is a convex polyhedron in the Euclidean n^2-space and whose vertices are the $n \times n$ permutation matrices. Suppose S_n is the symmetric group of degree n, and $\chi : S_n \to \mathbb{C}$ is an irreducible character of S_n with degree greater than 1. Consider the immanant d_χ defined by

$$d_\chi(X) = \sum_{\sigma \in S_n} \chi(\sigma) \prod_{j=1}^{n} X_{j,\sigma(j)}, \qquad X = (X_{jk}) \in M_n.$$

It is clear that if the degree of the character χ is one, then d_χ is the determinant or the permanent.

The behavior of the permanent on Ω_n has been studied extensively. M. Marcus and M. Newman proved that

$$perS \leq 1,$$

for all $S \in \Omega_n$, and $perS = 1$ if and only if $S = P(\sigma)$, for some $\sigma \in S_n$.

Let T be a linear map from Ω_n into Ω_n, such that

$$T(\lambda S_1 + (1-\lambda)S_2) = \lambda T(S_1) + (1-\lambda)T(S_2),$$

for all $S_1, S_2 \in \Omega_n$ and for all real numbers λ, $0 \leq \lambda \leq 1$. We say that the linear map T preserves d_χ if $d_\chi((T(S)) = d_\chi(S)$ for all $S \in \Omega_n$.

The linear mappings which preserve the permanent on Ω_n are already characterized.

We characterize the linear surjective maps T defined in Ω_n that preserve the immanant d_χ, where the character χ has degree greater than one.

Keywords: immanants, linear preserver problems, doubly stochastic matrices.

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On linear maps that convert singular matrices to matrices in the zero set of an immanant

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Abstract

Let χ be an irreducible character of the symmetric group of degree n and let d_χ be the immanant associated with χ. We prove that if $n \geq 3$, there is no linear transformation $T : M_n(\mathbb{C}) \to M_n(\mathbb{C})$ satisfying

$$\det(X) = 0 \Leftrightarrow d_\chi(T(X)) = 0,$$

for all $X \in M_n(\mathbb{C})$

Keywords: linear preserver problems, determinant, immanant.

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Conditions for a decomposable symmetric tensor associated with a spherical function to be zero

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Abstract

Let V a finite dimensional vector space over the complex numbers. Let N be a positive integer with $N \geq 2$. Let $\bigotimes^N V$ be the Nth tensor power of V, and $x_1 \otimes \cdots \otimes x_N$ the tensor product of the vectors $x_1, \ldots, x_N$. Let S_N be symmetric group of degree N. For each $\sigma \in S_N$ there exists a unique linear mapping $P(\sigma) : \bigotimes^N V \longrightarrow \bigotimes^N V$ such that

$$P(\sigma)(x_1 \otimes \cdots \otimes x_N) = x_{\sigma^{-1}(1)} \otimes \cdots \otimes x_{\sigma^{-1}(N)}$$

for all $x_i \in V$, $i = 1, \ldots, N$. If λ is a complex valued function of S_N we denote by $T(S_N, \lambda)$ the operator

$$T(S_N, \lambda) = \frac{\lambda(id)}{N!} \sum_{\sigma \in S_N} \lambda(\sigma) P(\sigma)$$

where id denotes the identity element of S_N.

Let $\lambda = (\lambda_1, \ldots, \lambda_q)$ be a partition of N. We denote the partition and the character it induces in S_N by the same letter λ. We define in the set of all partitions of N the partial order: If $\alpha = (\alpha_1, \ldots, \alpha_t)$, $\beta = (\beta_1, \ldots, \beta_s)$ are partitions of N then

$$\alpha \prec \beta \Leftrightarrow s \leq t \wedge \sum_{i=1}^{v} \alpha_i \leq \sum_{i=1}^{v} \beta_i \ , \ \forall v = 1, \ldots, s.$$

Let m and p be a positive integers with $m < p$. We identify S_m with the subgroup $\{\sigma \in S_p : \sigma(j) = j \ , \forall j = m+1, \ldots, p\}$ of S_p. Let λ (respectively χ) be an irreducible compex character of S_p (repectively S_m).

The spherical function $\varphi_{\lambda,\chi}$ is a complex valued function of S_p defined by

$$\varphi_{\lambda,\chi}(g) = \frac{\lambda(id)\chi(id)}{m!p!} \sum_{h \in S_m} \lambda(gh)\chi(h^{-1}) \ , \ g \in S_p.$$

We denote by $(\lambda, \chi)_{S_m}$ the nonnegative integer

$$(\lambda, \chi)_{S_m} = \frac{1}{m!} \sum_{h \in S_m} \lambda(h)\chi(h^{-1})$$

and by A_λ the set

$$A_\lambda = \{\chi \in Irr(S_m) : (\lambda, \chi)_{S_m} \neq 0\}$$

where $Irr(S_m)$ denotes the set of all irreducible characters of S_m.

Let χ be a minimal element of A_λ relatively to the partial order $\prec$. A necessary and sufficient condition on the vectors $x_1 \otimes \cdots \otimes x_p$ is given for $T(S_p, \varphi_{\lambda,\chi})(x_1 \otimes \cdots \otimes x_p)$ to be zero.

Frobenius endomorphisms and the determinantal range

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Abstract

Let A and C be square complex matrices of size n, the set

$$W_C(A) = \{\mathrm{Tr}(AUCU^*) : \ UU^* = I_n\}$$

is the $C-$numerical range of A and it reduces to the classical numerical range, when C is a rank one Hermitian orthogonal projection. A variation of $W_C(A)$ is the $C-$determinantal range of A, that is,

$$\triangle_C(A) = \{\det(A - UCU^*) : UU^* = I_n\}.$$

We present some properties of this set and characterize the additive Frobenius endomorphisms for the determinantal range on the whole matrix algebra M_n and on the set of Hermitian matrices H_n.

Keywords: C-Determinantal range, Frobenius endomorphisms, σ-points, real sets.

IMS8

PART I: Statistical inference

Organizer: Luís Miguel Grilo (Portugal)

A new approximation to the product distribution of beta independent and identically distributed random variables

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Abstract

Asymptotic expansions of ratios of gamma functions play a key role in the achievement of approximated distributions for some likelihood ratio test statistics used in Multivariate Analysis, namely those whose distribution has been shown to be that of the product of a number of independent beta random variables. The proposed asymptotic expansion of the ratio of two gamma functions leads to an accurate and highly manageable new asymptotic distribution of the product of independent and identically-distributed beta random variables. In this framework, a new approximation to the Wilks Lambda statistic distribution is proposed and numerically assessed.

Keywords: expansions of ratios of Gamma functions, mixtures of gamma distributions, generalized near-integer gamma distribution.

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Hierarchical cluster analysis of cyanophyta phytoplankton variables in Dammed Water Bodies

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Abstract

The dataset used in this study contains biological (phytoplankton) variables of the water column of several Portuguese reservoirs. This dataset was collected from measurements of the Laboratory of Environment and Applied Chemistry (LABELEC) [Cabecinha et al., 2009] to evaluate the impact of local and regional scale factors on the phytoplankton community structure of dammed water bodies. The original data refers to seasonal samples taken between 1995 and 2005 (4 samples per year) for each reservoir. In this study, the average values for each reservoir and each season were considered. The analyzed dataset consisted of 257 variables and 135 cases.

Cluster Analysis has been recognized as a powerful statistical tool to reduce data when a large number of records is available. By grouping the objects of the study in *clusters*, defined by the characteristics present in the selected variables, the records in each cluster have similar behavior, allowing to assign a new meaning for the objects in each cluster. Usually, cluster analysis is applied to obtain clusters of cases, but it also can be applied in order to obtain clusters of variables. A thorough cluster analysis, using R software and exploring several additional packages, was applied to the phytoplankton variables, to find associations between the records in the dataset and to find new interpretations for groups of reservoirs [Correia et al., 2014]. These reservoirs were divided into three clusters: (1) Interior Tagus and Aguieira; (2) Douro; and (3) Other rivers.

In this work, we apply hierarchical cluster analysis to the 38 phytoplankton variables of the phylum Cyanophyta, based on records corresponding to Summer period. The choice of these types of algae was due to their dangerous characteristics for public health, namely in Summer, when the reservoirs are used by the public for swimming and leisure. Three clusters of Cyanophyta variables were found.

Using ANOVA, Kruskal Wallis and Tukey tests, we compared the means and medians of the now obtained Cyanophyta clusters for the reservoirs belonging to Interior Tagus and Aguieira, Douro and Other rivers, in order to validate the classification of the water quality of reservoirs. We found that the amount of Cyanophyta algae present in the reservoirs from the three clusters is significantly different.

Keywords: multivariate statistical analysis, environmental data, water quality, reservoirs, ANOVA.

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Stability of the relative relevances

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Abstract

The models we consider are based on the spectral decomposition of the mean matrices for matrices with degree $k \geq 1$. In a previous work we studied the relative relevance of the spectral component associated to the first eigenvalue in situations where this eigenvalue is strongly dominant. A stability analysis of our results when the second eigenvalue increases relatively to the first is carried out.

Keywords: stochastic matrices, eigenvalues, stability, model degree.

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Point estimation in mixed models

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Abstract

The estimation of variance components and estimable vectors is carried out for mixed models $\boldsymbol{Y} = \sum_{i=0}^{w} \boldsymbol{X}_i \boldsymbol{\beta}_i$, where $\boldsymbol{\beta}_0$ is fixed and $\boldsymbol{\beta}_1, ..., \boldsymbol{\beta}_w$ are random. We assume the $\boldsymbol{\beta}_1, ..., \boldsymbol{\beta}_w$ to have null mean vectors and null cross covariance matrices having variance-covariance matrices $\sigma_1 \boldsymbol{I}_{c_1}, ..., \sigma_w \boldsymbol{I}_{c_w}$. Orthogonality features given by commutation of matrices $\boldsymbol{M}_i = \boldsymbol{X}_i \boldsymbol{X}_i^{'}, i = 1, ..., w$, and $\boldsymbol{T}$, the orthogonal matrix on the space spanned by the mean vector, $\boldsymbol{\mu} = \boldsymbol{X}_0 \boldsymbol{\beta}_0$, is progressively introduced. The improvement of the estimators following the increase of orthogonality is shown. The unification achieved rests on an algebraic result established in this work.

Keywords: inducing pivot variables, variance components, measurable functions.

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Global confidence regions for mixed models assuming orthogonal block structure and normality

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Abstract

Given a mixed model

$$\boldsymbol{Y} = \sum_{i=0}^{w} X_i \boldsymbol{\beta}_i$$

with $\boldsymbol{\beta}_0$ fixed and $\boldsymbol{\beta}_1, ..., \boldsymbol{\beta}_w$ independent with null mean vectors and variance covariance matrices $\theta_1 I_{c_1}, ..., \theta_w I_{c_w}$ we intend to derive $1-q$ level global regions. These will contain with probability $1-q$ any future observation whenever $\boldsymbol{\beta}_0$ and $\boldsymbol{\theta} = (\theta_1 ... \theta_w)$ are the same as for the model.

In deriving these regions we assume that $\boldsymbol{Y}$ is normal and has orthogonal block structure.

Keywords: OBS, UMVUE, variance components.

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The exact distribution of the total median and the total range statistics

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Abstract

The classic procedures of inference developed under the assumption that the observations come from a normal population may be inappropriate if we have other distributions or some disturbances in the data. It is thus important to find efficient and robust estimators of location and scale parameters, in particular, for use in statistical quality control and reliability. Here we focus on the total median, TMd, and the total range, TR, statistics, defined in [Cox and Iguzquiza, 2001], [Figueiredo and Gomes, 2001] and [Figueiredo, 2003a, 2003b]. These estimators are related to the bootstrap sample associated with an observed sample, and after some combinatorial computations, they can be written as a weighted mean of the sample order statistics. In [Figueiredo, 2003a, 2003b] and [Figueiredo and Gomes, 2004, 2006], we analyzed their efficiency and we compared their robustness with the one obtained for the traditional estimators of location and spread. Here we derive the exact distribution of these statistics for some models and several sample sizes. For the exponential distribution the TMd and the TR statistics can be written as a mixture of independent exponential variables or as a linear combination of independent chi-square variables, and following [Box , 1954] and [Dempster and Kleyle, 1968] we get the exact distribution of these statistics. In general it is not possible to obtain the exact, or at least a manageable, distribution for these statistics. Thus, as a future work we aim to obtain near-exact distributions for these statistics, along the line of the works of [Coelho, 2004], [Grilo, 2005] and [Grilo and Coelho, 2007, 2011], and to compare these approximations with the corresponding exact and simulated distributions.

Keywords: bootstrap sample, order statistics, near-exact distributions, total median statistic, total range statistic.

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Hadamard matrices on information theory

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Abstract

Hadamard matrices give rise to a numerous and diverse set of applications, beginning with applications in experimental design theory and the theory of error-correcting codes, until important applications in quantum information, communications, networking and cryptography that were found unexpectedly. The existence of Hadamard matrices remains one of the most challenging open questions in combinatorics. Substantial progress on their existence has resulted from advances in algebraic design theory using deep connections with linear algebra, abstract algebra, finite geometry, number theory, and combinatorics. The construction and analysis of Hadamard matrices, and their use on combinatorial designs, play an important role nowadays in diverse fields such as: quantum information, communications, networking, cryptography, biometry and security. Hadamard Matrices are present in our daily life and they give rise to a class of block designs named Hadamard configurations. It is easy and current to find different applications of it based on new technologies and codes of figures such as Quick Response Codes (QR Codes). Balanced Incomplete Block Designs (BIBD) are very well known as a tool to solve emerging problems in this area. In this work types of new combinatorial designs, External Difference Families (EDF), External BIBD (EBIBD) and Splitting BIBD (SBIBD) will be explored with illustrations on their applications to authentication codes and secret sharing schemes secure against hackers. Applications of these Combinatorial Designs to Authentication Codes and robust secret sharing schemes will be presented, as well as the rule of using R software in this issue.

Keywords: authentication Code, BIBD, combinatorial design, QR codes, R software.

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The exact and near-exact distributions for the statistic used to test the reality of covariance matrix in a complex normal distribution

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Abstract

We start to express the exact distribution of the negative logarithm of the likelihood ratio statistic, used to test the reality of the covariance matrix in a certain complex multivariate normal distribution, as an infinite mixture of Generalized Near-Integer Gamma (GNIG) distributions. Based on this representation we develop a family of near-exact distributions for the likelihood statistic, which are finite mixtures of GNIG distributions and match, by construction, some of the first exact moments. Using a proximity measure and for some family members we illustrate the excellent and well-known properties of the near--exact distributions. They are very close to the exact distribution but far more manageable and with very good asymptotic properties for increasing sample sizes and also for increasing number of variables. The near-exact distributions are even much more accurate than the asymptotic approximation considered, namely when the sample size is small and the number of variables involved is large. The corresponding cumulative distribution functions allow us an easy computation of very accurate near-exact quantiles.

Keywords: characteristic function, Beta distribution, Gamma distribution, small samples, quantiles.

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A principal components analysis of environmental variables of the water column in reservoirs

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Abstract

In this paper we present a principal components analysis of chemical and environmental variables of the water column (such as temperature, pH, CO2, Dissolved Oxygen, etc.) and hydromorphological features of several Portuguese reservoirs, to reduce the number of variables.

The dataset consisted of 257 variables and 135 cases, collected from measurements of the Laboratory of Environment and Applied Chemistry (LABELEC) referring to seasonal samples taken between 1995 and 2005 (4 samples per year for each reservoir) and it was used in [Cabecinha et al., 2009] and [Correia et al., 2014] for deriving a classification of reservoirs according to their water quality. The environmental variables were collected at 100 m from the reservoirs' crest, at two different depths: a) near the surface, approximately 0.5 m depth (*epilimnion*); and b) near the bottom, 2 m above bottom (*hypolimnion*). The epilimnion is the top-most layer in a thermally stratified lake, occurring above the deeper hypolimnion. It is warmer and typically it has a higher pH and higher dissolved oxygen concentration than the hypolimnion. Because this layer receives the most sunlight it contains the most phytoplankton. As they grow and reproduce, they absorb nutrients from the water; when they die, they sink into the hypolimnion, resulting in the epilimnion becoming depleted of nutrients. In this study, the average values for each reservoir and each season were considered.

Using the principal components method, the environmental variables measured in the epilimnion and in the hypolimnion, together with the hydromorphological characteristics of the dams were reduced from 63 variables to only 13 components, which explained a total of 83.348% of the variance in the original data. After rotation with VARIMAX method was performed, the relations between the principal components and the original variables got clearer and more explainable, which provided a factor analysis model for these environmental variables using 13 factors, such as: *Water quality and distance to the source, Hypolimnion chemical composition, Sulfite-reducing bacteria and nutrients, Coliforms and faecal streptococci, Reservoir depth, Temperature, Location*, among other factors.

Keywords: multivariate statistical analysis, factor analysis, water quality, epilimnion, hypolimnion.

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Mixed models with random sample sizes: Observations failures

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Abstract

Analysis of variance (ANOVA) is routinely used in several areas, such as in Medicine, Agriculture or Phycological, where the sample sizes may not be previously known. This often occurs when there is a fixed time span for collecting the observations. A good example is the collecting data from patients with several pathologies arriving at a hospital during a fixed time period (see e.g. [Moreira et al., 2013] and [Nunes et al., 2014]). Another important case arises when one of the pathologies is rare since, in that case, the number of patients in the sample set may not be achieved (see [Nunes et al., 2012]).

In these situations, assuming there are m different treatments, it is more correct to consider the sample sizes as realizations, $n_1, ..., n_m$, of independent random variables, $N_1, ..., N_m$ (see e.g. [Mexia et al., 2011] and [Nunes et al., 2012, 2014]). This approach must be based on an adequate choice of the distribution of $N_1, ..., N_m$. We may assume the Poisson distribution when the occurrence of observations corresponds to a counting process (see e.g. [Mexia et al., 2011] and [Nunes et al., 2014]). It is important to note that in this case there is no upper bounds for the sample sizes. Different is the situation in which such upper bounds exist. For instance, we may have the upper bounds, $r_1, ..., r_m$, which are not always attained, since failures may occur. Thus the Binomial distribution is a reasonable choice (see [Nunes et al., 2015]).

In the mentioned papers fixed effects ANOVA were approached. Now we intend to extend the results to mixed models, assuming that $N_1, ..., N_m$ are Binomial distributed with parameters $r_1, ..., r_m$ and $1 - p$, where p denote the probability of a failure, $N_i \sim \mathcal{N}(r_i, 1-p)$, $i = 1, ..., m$.

When approaching linear models through Commutative Jordan Algebras (CJA), the L extensions have been used to consider certain issues causing non-orthogonality in fixed and mixed effects models (see [Ferreira et al., 2009] and [Moreira et al., 2009]). Since the formulation of mixed models with random sample sizes gets easier when using L extensions, we will consider this class of models.

The applicability of the proposed approach is illustrated through an application on real medical data from patients affected by cancer in Brazil.

Keywords: ANOVA, mixed models, unknown sample sizes, binomial distribution, cancer registries.

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Joining segregated models with commutative orthogonal block structure

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Abstract

A mixed model

$$\mathbf{y} = \sum_{i=0}^{w} \mathbf{X}_i \boldsymbol{\beta}_i \ ,$$

where $\boldsymbol{\beta}_0$ is fixed and $\boldsymbol{\beta}_1, ..., \boldsymbol{\beta}_w$ are independent random vectors with null mean vectors and variance-covariance matrices $\theta_1 I_{c_1} ... \theta_m I_{c_m}$ where $c_i = rank(\mathbf{X}_i)$, $i = 1, ..., k$, and null cross-covariance matrices, has mean vector $\boldsymbol{\mu} = \mathbf{X}_0 \boldsymbol{\beta}_0$ and variance-covariance matrix $\mathbf{V}(\theta) = \sum_{i=1}^{w} \sigma_i^2 \mathbf{M}_i$ where $\mathbf{M}_i = \ \mathbf{X}_i \mathbf{X}_i^\tau$, $i = 1, ..., w$.

When the variance-covariance matrix can be written as

$$\mathbf{V}(\gamma) = \sum_{j=1}^{m} \gamma_j \mathbf{Q}_j \ ,$$

a linear combination of known pairwise orthogonal orthogonal projection matrices, POOPM, $\mathbf{Q}_1, ..., \mathbf{Q}_m$, that up to the identity matrix, and furthermore the orthogonal projection matrix, on the space spanned by $\boldsymbol{\mu}$, commutes with the orthogonal projection matrices in the expression of the variance-covariance matrix, the model is COBS (model with commutative orthogonal block structure).

Model joining enable us to build up complex models from simple ones, overlaping observations vectors obtained separately, in order to perform their joint analysis. The technic used to join COBS rests on their algebraic structure and the Cartesian product of commutative Jordan algebras.

A COBS has segregation when its random effects part is segregated as a sub-model. Since this property leads to interesting results for the estimation of variance components, we prove that joining COBS with segregation we obtain a new COBS with segregation.

Keywords: COBS, Jordan algebra, mixed model, model joining, segregation.

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Multivariate time series analysis: a study on the relation between imports, exports and economic growth

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Abstract

The world does not consist of independent stochastic processes. In fact, there are several phenomena, from economics (see e.g. [Ugur, 2008]) to neuroscience (see e.g. [Michalareas et al., 2013]), which can be described by multivariate time series that exhibit causal relationships. Here we start by giving an overview of the existent techniques for analyzing the behavior of multivariate time series, with particular emphases to the econometric framework. A study on the casual relation between imports, exports and economic growth in Portugal, is presented next. The data was obtained in *PORDATA, Base de Dados de Portugal Contemporâneo* and *INE, Instituto Nacional de Estatística*. Multivariate vector autoregressive (VAR) models are used since they have proven to be especially useful for describing and forecasting the behavior of econometric time series.

Keywords: multivariate statistics analysis, causality, stationarity, vector autoregressive models, international trade, gross domestic product, Portugal.

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Inference for bivariate integer-valued moving average models

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Abstract

Time series of (small) counts are common in practice and appear in a wide variety of fields. In the last three decades, their statistical analysis has emerged as an important area of research by exploring models that explicitly account for the discreteness of the data. Among the proposed models are the INARMA (INteger-valued AutoRegressive Moving Average) models, which are constructed by replacing the multiplication in the conventional ARMA models by an appropriate random operator. The most popular of such operator is the binomial thinning operator (Steutal and Van Harn, 1979). Scotto *et al* (2015) present an overview about univariate time series of counts. However, for multivariate time series of counts several difficulties arise and the literature is not so detailed.

This work presents the so called Bivariate INteger-valued Moving Average model of first order, BINMA(1, 1), proposed by Torres *et al.* (2012). The main probabilistic and statistical properties of BINMA models are exhibited. Emphasis is placed on models with Bivariate Poisson and Bivariate Negative Binomial distributions for the innovation process (Kocherlakota and Kocherlakota, 1992).

The generalized method of moments is used to estimate the parameters (Silva *et al.*, 2014). Finally, methods for model diagnostic and validation based on residual analysis, predictive distributions and parametric resampling methods are presented [Silva et al., 2015]. These methods will be illustrated by simulation and by using a real dataset.

Keywords: count time series, BINMA model, parameter estimation, model diagnostic.

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IMS8

PART II: Numerical and combinatorial methods

Organizer: Fernando Lucas Carapau (Portugal)

Markov chains and mechanical systems

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Abstract

We study, in the context of discrete dynamical systems, a method of associating Markov chains to certain chaotic mechanical systems. We analyze in particular a chaotic pendulum. We study systematically, through PoincarÃľ sections, discretizations which maintain the most relevant properties of the system. Therefore, we transform a differential dynamical system to a discrete dynamical system. For certain values of the parameters there exist one or more attractors. In this case, to the discrete system we associate a Markov topological chain. The dynamical properties of the original mechanical system can be characterized through the Markov chain and appropriate transition matrices.

Keywords: Markov chains, transition matrices, chaotic systems, pendulum, symbolic dynamics.

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Some results on the Frobenius coin problem

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Abstract

Let $\mathbb{N}$ denote the set of nonnegative integers. A numerical semigroup is a subset S of $\mathbb{N}$ closed under addition, it contains the zero element and has finite complement in $\mathbb{N}$. Given a nonempty subset A of $\mathbb{N}$ we will denote by $\langle A\rangle$ the submonoid of $(\mathbb{N},+)$ generated by A, that is,

$$\langle A\rangle = \{\lambda_1 a_1 + \cdots + \lambda_n a_n \mid n \in \mathbb{N}\backslash\{0\}, a_i \in A, \lambda_i \in \mathbb{N} \text{ for all } i \in \{1,\ldots,n\}\}.$$

It is well known that $\langle A\rangle$ is a numerical semigroup if and only if $\gcd(A) = 1$. If S is a numerical semigroup and $S = \langle A\rangle$ then we say that A is a system of generators of S. Moreover, if $S \neq \langle X\rangle$ for all $X \subsetneq A$, then we say that A is a minimal system of generators of S. It is well known that every numerical semigroup admits a unique minimal system of generators, which in addition is finite. The cardinality of its minimal system of generators is called the embedding dimension of S, denoted by $\mathrm{e}(S)$. Following a classic line, two invariants have special relevance to a numerical semigroups: the greatest integer that does not belong to S, called the Frobenius number of S denoted by $\mathrm{F}(S)$, and the cardinality of $\mathbb{N}\backslash S$, called the gender of S denoted by $\mathrm{g}(S)$. The Frobenius coin problem (often called the linear Diophantine problem of Frobenius) consists in finding a formula, in terms of the elements in a minimal system of generators of S, for computing $\mathrm{F}(S)$ and $\mathrm{g}(S)$. This problem was solved by Sylvester for numerical semigroups with embedding dimension two. Sylvester demonstrated that if $\{n_1, n_2\}$ is a minimal system of generators of S, then $\mathrm{F}(S) = n_1n_2 - n_1 - n_2$ and $\mathrm{g}(S) = \frac{1}{2}(n_1-1)(n_2-1)$. The Frobenius coin problem remains open for numerical semigroups with embedding dimension greater than or equal to three. In this talk we will present some classes of numerical semigroups for which this problem is solved (see References).

Keywords: numerical semigroup, Frobenius number, embedding dimension, Genus, Mersenne numbers.

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One-dimensional model of fluids of third grade in straight tubes with constant radius

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Abstract

In recent years the Cosserat theory approach has been applied in the field of fluid dynamics to reduce the full three-dimensional system of equations of the flow motion into a one-dimensional system of partial differential equations which, apart from the dependence on time, depends only on a single spatial variable. Applying this approach theory in the particular case of a straight tube of constant circular cross-section, we obtain a one--dimensional model related with the flow of a viscoelastic fluid of differential type with complexity $n = 3$. From this reduced system, we derive unsteady equations for the wall shear stress and mean pressure gradient depending on the volume flow rate, tube geometry, Womersley number and viscoelastic coefficients over a finite section of the straight rigid tube. Attention is focused on some numerical simulations of unsteady flow regimes.

Keywords: one-dimensional model, viscoelastic fluid, unsteady flow, hierarchical theory, Cosserat theory.

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Numerical semigroups and interval maps

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Abstract

Interval maps constitute a very important class of discrete dynamical systems with a well developed theory. Our purpose is to study a particular class of interval maps for which the set of periods is a numerical semigroup. Our approach is twofold: given a family of interval maps, parametrized by a certain set of parameters, construct a family of numerical semigroups corresponding to the existing periods in the given family of maps, and the reverse, obtain a family of interval maps which realize a given family of semigroups as their period set.

Keywords: Markov maps, unidimensional maps, numerical semigroups, periodic points, Markov matrix.

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Complete synchronization and delayed synchronization for different couplings

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Abstract

We consider a general coupling of two identical chaotic dynamical systems and we obtain the conditions for synchronization. We consider two types of synchronization: complete synchronization and delayed synchronization. Then, we consider four different couplings having different behaviors regarding their ability to synchronize either completely or with-delay: Symmetric Linear Coupled System, Commanded Linear Coupled System, Commanded Coupled System with Delay and Symmetric Coupled System with Delay. The values of the coupling strength for which a coupling synchronizes define its Window of Synchronization. We obtain analytically the Windows of Complete Synchronization and we apply it for the considered couplings that admit complete synchronization. We also obtain analytically the Window of Chaotic Delayed Synchronization for the only considered coupling that admits a chaotic delayed synchronization, the Commanded Coupled System with Delay. At last, we use four different free chaotic dynamics (based in tent map, logistic map, three piecewise-linear map and cubic like map) in order to observe numerically the analytically predicted windows.

Keywords: synchonization, chaotic systems.

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On a modified log-conformation formulation of traceless variant of Oldroyd–B viscoelastic model

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Abstract

This work describes a modified log-conformation of traceless variant of Oldroyd–B viscoelastic model and it follows the recentt work developed by T. Bodnar, M. Pires and J. Janela about the traceless variant of Johnson-Segalman viscoelastic model and the modified log-conformation formulation of viscoelastic fluid flows proposed by P.Saramito. The numerical simulation of viscoelastic fluid flows based on Oldroyd type models, in particular Oldroyd–B models are often sensible to numerical instabilities at higher Weissenberg numbers, recognized by High Weissenberg Number Problem. All numerical methods break down when Weissenber number reaches the critical value. The maximum Weissenberg number is dependent of mesh, method and the constitutive model. A possible source for numerical instabilityies can be due to exponential growth of the stress with convection. The flows with low-Weissenberg number, Newtonian or quasi-Newtonian models as well corotational models don't exhibit such instabilities and they have at common the fact that all of these flows are traceless. Them are the motivation to traceless variant of Oldroyd–B viscoelastic model. By conjugating the traceless variant with the modified log-conformation we expect to obtain higher maximal Weissenberg number. This work resumes the efforts in this sense.

Keywords: Heigh Weissenberg number, numerical instabilities, Oldroyd–B fluid, operator-splitting, matrix logarithm.

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Growing stochastic matrices and ontogenesis of dynamical systems

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Abstract

We present several methods to study discrete dynamical systems which change with time. In particular, we define and characterize semigroup actions on the space of Markov discrete dynamical systems (MDDS), through transition matrices. A MDDS is, in our context, seen as an elementary object, as the state of a larger dynamical system. We define a class of endomorphims on the space MDDS which are determined by block matrices endomorphims. A particular dynamical system defined on the space MMDS is determined by a given set of endomorphisms, which generate the evolution semigroup. The time evolution of a MDDS is then seen as a path in the Cayley graph of the given semigroup. Appropriate measures of the complexity of the global system are defined and calculated. This method allow us to study systematically changing dynamical systems - processes called ontogenesis - and the evolution of populations of dynamical systems - processes called phylogenesis.

Keywords: Markov systems, transition matrices, arbitrary dimension, evolution semigropus, ontogenesis.

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Hybrid chaotic systems

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Abstract

We consider piecewise defined differential dynamical systems which can be analysed through symbolic dynamics and transition matrices. We have a continuous regime, where the time flow is characterized by ODE which have explicit solutions, and the singular regime, where the time flow is characterized by an appropriate transformation. The symbolic codification is given through the association of a symbol for each distinct regular system and singular system. The transition matrices are then determined as linear approximations to the symbolic dynamics. We analyze the dependence on initial conditions, parameter variation and the occurrence of global strange attractors.

Keywords: dynamical systems, symbolic dynamics, iteration theory, transition matrices, attractors.

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IMS9

Statistical models with matrix structure

Organizer: Miguel Fonseca (Portugal)

To be announced

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Abstract

To be announced.

To be announced

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Abstract

To be announced.

Approximation with Kronecker product structure with one component as compound symmetry matrix

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Abstract

Statistical modeling of doubly multivariate data, such as e.g. space-time data, has often been based on separable covariance matrix, that is, covariances that can be written as a Kronecker product of a purely spatial covariance $\boldsymbol{\Psi} \in \mathbb{R}_{p\times p}$ and a purely temporal covariance $\boldsymbol{\Sigma} \in \mathbb{R}_{q\times q}$. In many applications it is assumed, that in this separable form of a covariance matrix one factor has a specific structure, such as e.g. compound symmetry correlation structure, where $\boldsymbol{\Psi} = (1-\rho)\mathbf{I}_p - \rho\mathbf{J}_p = \boldsymbol{\Psi}_{CS}$, with $1/(p-1) < \rho < 1$.

In statistical research of this type, often the approximation of non-separable doubly multivariate data covariance matrix can be applied, for example to calculate the power of the test for verifying the structure of the covariance matrix. Van Loan and Pitsianis (1992) and Genton (2007) described the nearest Kronecker product approximation of non--separable covariance matrix, using Frobenius norm. The aim of this paper is to solve the following problems:

1. If it is known that a given $pq \times pq$ matrix $\boldsymbol{\Omega}$ has $\boldsymbol{\Psi}_{CS} \otimes \boldsymbol{\Sigma}$ structure, how to determine the matrices $\boldsymbol{\Psi}_{CS}$ and $\boldsymbol{\Sigma}$?
2. If it is known that a given $pq \times pq$ matrix $\boldsymbol{\Omega}$ is not two-separable, how to find the matrices $\boldsymbol{\Psi}_{CS}$ and $\boldsymbol{\Sigma}$ such that matrix $\boldsymbol{\Psi}_{CS} \otimes \boldsymbol{\Sigma}$ provides the nearest approximation of $\boldsymbol{\Omega}$?

Keywords: separable structure, compound symmetry structure, Frobenius norm.

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Best unbiased estimators for doubly multivariate data

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Abstract

My presented presentation deals with the best unbiased estimators of the blocked compound symmetric covariance structure for $m-$variate observations over u sites under the assumption of multivariate normality and equal mean vector over sites. The free-coordinate approach is used to prove that the quadratic estimation of covariance parameters is equivalent to linear estimation with a properly defined inner product in the space of symmetric matrices. Using properties of Jordan algebra we can prove completeness of statistics and conclude that the estimators are best unbiased. Finally, strong consistency is proven. The properties of the estimators in the proposed model are compared with the ones of the model in Roy et al (2015). The proposed method is implemented with a real data set. The presentation is based on the paper send for publication by the authors: Arkadiusz Kozioł, Anuradha Roy, Roman Zmyślony, Ricardo Leiva, Miguel Fonseca. This presentation is connected with invited lecture by Zmyślony.

Keywords: best unbiased estimator, blocked compound symmetric covariance structure, doubly multivariate data, coordinate free approach, structured mean vector.

New concept of OBS for mixed models

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Abstract

Models with orthogonal block structure, OBS, would have variance-covariance matrices that were all positive semi-defined linear combinations of pairwise orthogonal orthogonal projection matrices $Q_1, \ldots, Q_m$ that add up to I_n.

When considering mixed models, we show that this requirement on variance-covariance matrices does not hold in general, and present an alternative in which those matrices would be the $\sum_{j=1}^{w} \gamma_j Q_j$ with γ_j spanning an open set contained in the family $R_{>}^{w}$ of w-dimensioned vectors with non-negative components.

When this new requirement holds and normality is assumed, we have UMVUE estimators for variance components and estimable vectors which will also be UBLUE.

Test for covariance matrix with use of spectral moments

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Abstract

Results regarding independence tests for a covariance matrix of size $p \times p$ will be presented. A new test is derived under the Kolmogorov condition $\frac{n}{p} \overset{n,p\to\infty}{\to} c$, where n denotes the sample size. Moments and cumulants that play a key role when deriving the distribution of the test statistics are obtained using a recursive formula given in [Pielaszkiewicz et al., 2015].

Keywords: goodness of fit test, covariance matrix, spectral moments, spectral cumulants.

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Contribute on mixed linear models-simultaneous diagonalization of the variance-covariance matrices

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Abstract

This work aim to introduce a new method of estimating the variance components in mixed linear models. The approach will be done firstly for models with 3 variances components and secondly attention will be devoted to general case of models with an arbitrary number of variance components.

In our approach, we construct and apply a finite sequence of sub-diagonalizations to the covariance structure of a given mixed linear model in order to produce a set of Gauss--Markov sub-models. Hypothesis tests and confidence intervals for the estimators achieved will be given.

Keywords: mixed linear model, variance components, orthogonal matrices, simultaneous diagonalization.

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Part VII

Contributed Talks

Total nonnegativity of matrices related to polynomial roots and poles of rational functions

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Abstract

In our talk we consider matrices which are related to stability of polynomials and to properties of the poles and zeros of rational functions. Specifically, in the case of polynomials we focus on matrices of Hurwitz type which are closely related to (Hurwitz) stability of a polynomial, i.e., to the property that all zeros are contained in the open left half of the complex plane. In the case of rational functions we consider Hankel matrices associated with the Laurent series at infinity of rational functions and focus on R-functions of negative type, i.e., functions which map the open upper half-plane of the complex plane to the open lower half-plane. For references and properties of this important class of functions we refer to the survey given in [Holtz and Tyaglov, 2012]. In the polynomial as well as in the rational case we are interested in *interval* problems which arise when the polynomial coefficients are due to uncertainty caused by, e.g., data uncertainties, but can be bounded in intervals. For background material from control theory and practical applications see [Barmish, 1994], [Bhattacharyya et al., 1995]. Specifically, we derive a sufficient condition for the Hurwitz stability of an interval family of polynomials and investigate in the case of R-functions the invariance of exclusively positive poles or exclusively negative roots in the presence of variation of the coefficients of the numerator and denominator polynomials within given intervals. It turns out that certain properties concerning the zeros and the poles remain in force through all the coefficient intervals if up to four polynomials of the entire family have certain properties. Typically, the coefficients of these polynomials alternate in attaining the endpoints of the coefficient intervals. This up-and-down behavior corresponds to a checkerboard pattern of the entries of the associated matrices.

The underlying property of all the matrices considered in this paper is that all their minors are nonnegative. Such matrices are called *totally nonnegative*. For properties of these matrices we refer to the monographs [Fallat and Johnson, 2011], [Pinkus, 2010]. In [Adm and Garloff, 2014] we derive an efficient determinantal test based on the Cauchon algorithm [Goodearl et al., 2011], [Launois and Lenagan, 2014] for checking a given matrix for total nonnegativity and related properties. In our talk we apply this test to the (infinite) matrices mentioned above. It turns out that properties of the infinite matrix can be inferred from properties of one or two finite sections of this matrix. To solve the related interval problems we make use of a result in [Adm and Garloff, 2013] by which from the nonsingularity and the total nonnegativity of two matrices we can infer that all matrices lying between these two matrices are nonsingular and totally nonnegative, too. Here "between" is meant in the sense of the checkerboard ordering.

Keywords: totally nonnegative matrix, totally positive matrix, Hurwitz matrix, Hankel matrix, R-function, interval polynomial.

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Structured matrices and high relative accuracy

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Abstract

Computations with high relative accuracy for some classes of structured matrices are presented. In the literature, it is only possible to find algorithms with high relative accuracy for special classes of matrices related with total positivity (cf. [Koev, 2007]) or with diagonal dominance (cf. [Ye, 2008], [Demmel and Koev, 2004], [Peña, 2004]). Furthermore, we only have algorithms to carry out some computations with the corresponding matrices. In this talk, we extend some of these algorithms for totally positive matrices to a more general class of matrices called SBD matrices. This class includes totally positive matrices as well as their inverses. In particular, given adequate parameters, we can compute eigenvalues, singular values and inverses of SBD matrices with high relative accuracy (cf. [Barreras and Peña, 2013a]). We also present algorithms with high relative accuracy to compute the LDU decomposition of two classes of structured matrices: diagonally dominant M–matrices and almost diagonally dominant Z–matrices (cf. [Barreras and Peña, 2012/13], [Barreras and Peña, 2013b]).

Keywords: high relative accuracy, totally positive matrices, M–matrices.

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On the componentwise product of totally non-negative, structured matrices generated by functions in the Laguerre-Pólya class

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Abstract

New infinite, totally non-negative (TNN) matrices were exhibited by Holtz and Tyaglov (2012). It was shown that real polynomials p with exclusively negative zeros and positive coefficients generate a certain structured, infinite matrix $H = H(p)$ which is TNN. The matrix H bears the Hurwitz structure, but its precise relation to the classical Hurwitz matrix of the stability problem was not determined. Dyachenko (2014) gave a complete characterization of those power series giving rise to a TNN matrix H of so-called "Hurwitz-type".

We show that H is the limit of Hurwitz matrices related to certain stability problems. This new observation leads to independent, short and simple proofs of results on TNN matrices for aperiodic polynomials and generalized positive pairs of these, and facilitates extensions to functions (and positive pairs) in the Laguerre-Pólya class $\mathcal{L}$-$\mathcal{P}^+$ with only positive non-trivial MacLaurin coefficients. The discovered approach moreover leads to the first results on the componentwise product of the considered structured matrices. We show that the Schur-Hadamard product $H_i \circ H_j$ of matrices generated by positive pairs of entire functions in the said Laguerre-Pólya class, is itself TNN.

Keywords: negative roots, Hurwitz matrix, Hadamard product.

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A numerical solution for a telegraph equation using Bernstein polynomials technique

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Abstract

In this paper, a numerical solution of a telegraph partial derivative equation throughout Bernestein polynomials is proposed. The telegraph equation, describes wave propagation of electric signals in a cable transmission line, is a one dimensional linear time--invariant hyperbolic partial derivative equation. We adopt Bernstein polynomials to solve the system of equations defined on the region of an unit cable transmission line length and a given time window of size T. These partial derivative equations with respect to the variable x are derived from the application of Rothe's time discretization scheme for a telegraph equation. The system of one dimensional partial derivative equations is transformed in a large sized diagonal matrix which can be viewed as the system of linear equations after dispersing the variable. Each element of the large sized diagonal matrix represents a Bernestein operational matrix at a given sample time. A numerical solution, by solving the linear system of algebraic equations, is obtained and its accuracy and complexity are discussed. Numerical experimentations are conducted to demonstrate the viability of Bernestain polynomials technique. It is also shown the impact of Bernstein polynomials' orders and relevant sample points on the efficiency and accuracy of the proposed method.

Keywords: Bernstein polynomials, telegraph equation, partial differential equation.

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Preservers of local invertibility and local spectra of matrices

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Abstract

Let $\mathcal{M}_n$ be the set of $n \times n$ complex matrices, and for a nonzero vector $e \in \mathcal{C}^n$ and $T \in \mathcal{M}_n$, let $\sigma_T(e)$ denote the local spectrum of T at e. Local spectra play a very natural role in automatic continuity and in harmonic analysis, for instance in connection with the Wiener-Pitt phenomenon. For background material from local spectral theory, as well as investigations and applications in numerous fields, we refer to the books [Aiena, 2004], [Laursen and Neumann, 2000] and [Müller, 2007]. In this talk we present new results concerning the description of nonlinear maps ϕ on $\mathcal{M}_n$ leaving invariant the local spectra at e of $T \bullet S$ for different kind of binary operations $\bullet$ on matrices such as the difference $T - S$, the sum $T + S$, the product TS and the triple Jordan product TST, in a sense that

$$\sigma_{\phi(T)\bullet\phi(S)}(e) = \sigma_{T\bullet S}(e), \quad (T, S \in \mathcal{M}_n).$$

It is shown that such maps are of standard forms. As variant problems, mappings on $\mathcal{M}_n$ that compress or expand the local spectrum of these operations of matrices at a fixed nonzero vector are described. Part of the obtained results, extending some former results, belong to [Bendaoud, 2013, 2015], and to the two joint works [Bendaoud et al., 2013, 2015] where the corresponding problems in the infinite dimensional case, i.e., the characterization of nonlinear transformations on the algebra $\mathcal{L}(X)$ of all bounded linear operators on a Banach space X that preserve the local spectral radius or that compress the local spectrum of operators at a fixed vector are also discussed. These results lead to local versions of the main results of [Bhatia et al., 1999], [Chan et al., 2007] and [Molnár, 2001].

In [Bendaoud et al., 2014], we characterize additive surjective maps ϕ on $\mathcal{L}(X)$ which preserve the local invertibility, i.e., which satisfy

$$0 \in \sigma_{\phi(T)}(x) \text{ if and only if } 0 \in \sigma_{\phi(T)}(x) \text{ for every } x \in X \text{ and } T \in \mathcal{L}(X).$$

Extensions of this result to the case of different Banach spaces are also established. As application, additive maps from $\mathcal{L}(X)$ onto itself that preserve the inner local spectral radius zero of operators are classified. In our talk, by strengthening the preservability condition, we consider the nonlinear preservers of local invertibility on $\mathcal{M}_n$ or on $\mathcal{L}(X)$, and we obtain characterizations for mappings with less smoothness assumptions on them and discuss some related open problems.

Keywords: local spectrum, local (inner) spectral radius, single-valued extension property, nonlinear preservers.

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Integer powers of certain complex pentadiagonal Toeplitz matrices

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Abstract

Let A_n be an $n \times n$ pentadiagonal Toeplitz matrix as

$$A_n := \begin{bmatrix} a & 0 & b & & & & \\ 0 & a & 0 & b & & & \\ c & 0 & a & 0 & \ddots & & \\ & c & \ddots & \ddots & \ddots & b & \\ & & \ddots & 0 & a & 0 & b \\ & & & c & 0 & a & 0 \\ & & & & c & 0 & a \end{bmatrix}$$

where $a \in \mathbb{C}$ and $b, c \in \mathbb{C} \setminus \{0\}$. In this paper, we obtain a general expression for the entries of the rth ($r \in \mathbb{Z}$ if n is even; $r \in \mathbb{N}$ if n is odd) powers of A_n matrix. Additionally, we have the complex factorizations of Fibonacci polynomials.

Keywords: pentadiagonal matrix, Toeplitz matrix, powers of matrix, Fibonacci polynomials, complex factorizations.

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A canonical construction for nonnegative integral matrices with given line sums

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Abstract

Let p be a positive integer and let $\mathcal{A}^{(p)}(R,S)$ be the class of nonnegative integral matrices with entries less than or equal to p, with row-sum partition R, and column-sum partition S .

In this paper we state a necessary and sufficient condition for $\mathcal{A}^{(p)}(R,S) \neq \emptyset$. This condition generalizes the well known Gale-Ryser theorem. We also present a canonical construction for matrices in $\mathcal{A}^{(p)}(R,S)$.

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Keywords: integral matrices with given lines, partition domination, algorithm.

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Permutation matrices, doubly stochastic matrices and their $L-$rays

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Abstract

Recently some projection-type problems have been investigated for matrices, motivated by the notion of $X-$ray in the area of discrete tomography. In this talk we present some new results of this type. Let M_n be the space of real $n \times n$ matrices. We investigate a linear transformation $\sigma : M_n \to \mathbb{R}^n$, called an *$L-$ray*, which is defined in terms of sums of the entries in the blocks of a certain "$L-$shaped" partition of the positions of a matrix $A \in M_n$. We find descriptions of the image of the classes of permutation matrices and doubly stochastic matrices under this map, and show connections to majorization theory.

The talk is based on the paper [Dahl].

Keywords: permutation matrix, doubly stochastic matrix, majorization.

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Dahl, G. L-rays of permutation matrices and doubly stochastic matrices, to appear in *Linear Algebra Appl.*.

New estimations for the inverse of some special block matrices in the Euclidean matrix norm

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Abstract

Several block matrix splitting iterative methods for solving systems generated by discretizing partial differential equations, such as the parallel block-wise matrix multisplitting and two-stage multi splitting iteration methods (see [Bai, 1995, 1999]) has been developed. For the convergence analysis of these methods it is very useful to know a good estimation of the norm of the matrix inverse. On the other hand, for the error analysis for any linear system of the form $Ax = b$, an estimation of the norm of the inverse of a matrix A play a crucial role. This was the main motivation for developing several estimations for $\|A\|_\infty$ for some special subclasses of block $H-$matrices in [Cvetković and Doroslovački, 2014]. Here we will consider some different classes of nonsingular matrices and present estimations for their inverse in the Euclidean norm. Numerical experiments will illustrate the usefulness of new upper bounds in Euclidean norm.

Keywords: block matrices, Euclidean matrix norm, inverse matrix.

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Powers of certain complex tridiagonal matrices

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Abstract

In this paper, we obtain a general expression for the entries of the rth ($r \in \mathbb{Z}^+$) power of a certain $n \times n$ complex tridiagonal matrices as following

$$\begin{bmatrix} b & 0 & & & & & \\ a & b & a & & & & \\ & a & b & a & & & \\ & & \ddots & \ddots & \ddots & & \\ & & & a & b & a & \\ & & & & a & b & a \\ & & & & & 2a & b \end{bmatrix}.$$

In addition, we have the complex factorizations of Fibonacci polynomials.

Keywords: tridiagonal matrix, Chebyshev polynomial, matrix powers, eigenvalues, eigenvectors, Fibonacci polynomial, complex factorizations.

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Connecting sufficient conditions for the symmetric nonnegative inverse eigenvalue problem

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Abstract

We say that a list of real numbers is *symmetrically realisable* if it is the spectrum of some (entrywise) nonnegative symmetric matrix. The Symmetric Nonnegative Inverse Eigenvalue Problem (SNIEP) is the problem of characterising all symmetrically realisable lists.

We present a recursive method for constructing symmetrically realisable lists, based on a construction of Šmigoc (2004). The properties of the realisable family we obtain allow us to make several novel connections between a number of sufficient conditions developed over forty years, starting with the work of Fiedler in 1974. Specifically, we consider two methods of constructing symmetrically realisable lists: one due to Soules (1983) and later generalised by Elsner, Nabben and Neumann (1998), and one due to Soto (2013). We also consider a condition due to Borobia, Moro and Soto (2008) called *$C-$realisability* which is sufficient for the existence of a not-necessarily-symmetric realising matrix.

Using our recursive method, we show that the symmetrically realisable lists obtainable by Soules and Soto are identical and that these are precisely the $C-$realisable lists. In fact, we show that essentially all previously known sufficient conditions are either contained in or equivalent to the family we are introducing. As a corollary, we see that $C-$realisability is also sufficient for the symmetric problem. By viewing these lists through the lens of our recursive method, several interesting properties are also revealed.

Keywords: nonnegative matrices, symmetric nonnegative inverse eigenvalue problem, Soules matrix.

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Sign regular matrices having the interval property

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Abstract

We say that a class $\mathcal{C}$ of n-by-n matrices possesses the *interval property* if for any n-by-n interval matrix $[A] = [\underline{A}, \overline{A}] = ([\underline{a}_{ij}, \overline{a}_{ij}])_{i,j=1,\ldots,n}$ the membership $[A] \subseteq \mathcal{C}$ can be inferred from the membership to $\mathcal{C}$ of a specified set of its vertex matrices; here a *vertex matrix of* $[A]$ is a real matrix $B = (b_{ij})_{i,j=1,\ldots,n}$ with $b_{ij} \in \left\{\underline{a}_{ij}, \overline{a}_{ij}\right\}$ for all $i, j = i, \ldots, n$. Examples of such classes include the

- $M-$matrices or, more generally, inverse-nonnegative matrices [Kuttler, 1971], where only the bound matrices $\underline{A}$ and $\overline{A}$ are required to be in the class;
- inverse $M-$matrices [Johnson and Smith, 2002], where all vertex matrices are needed;
- positive definite matrices [Bialas and Garloff, 1984], [Rohn, 1994], where a subset of cardinality 2^{n-1} is required (here only symmetric matrices in $[A]$ are considered).

A class of matrices which in the nonsingular case are somewhat related to the inverse nonnegative matrices are the totally nonnegative matrices. A real matrix is called *totally nonnegative* if all its minors are nonnegative. Such matrices arise in a variety of ways in mathematics and its applications, e.g., in differential and integral equations, numerical mathematics, combinatorics, statistics, and computer aided geometric design. For background information we refer to the recently published monographs [Fallat and Johnson, 2011], [Pinkus, 2010]. The first author posed in 1982 the conjecture that the set of the nonsingular totally nonnegative matrices possesses the interval property, where only two vertex matrices are involved [Garloff, 1982], see also [Fallat and Johnson, 2011, Section 3.2] and [Pinkus, 2010, Section 3.2]. The two vertex matrices are the bound matrices with respect to the checkerboard ordering which is obtained from the usual entry-wise ordering in the set of the square matrices of fixed order by reversing the inequality sign for each entry in a checkerboard fashion. In our talk we apply the Cauchon algorithm [Adm and Garloff, 2014] (also called deleting derivation algorithm [Goodearl et al., 2011] and Cauchon reduction algorithm [Launois and Lenagan, 2014]) to settle the conjecture. We also obtain the result that a fixed zero-nonzero pattern of the minors stays unchanged through an interval of nonsingular totally nonnegative matrices.

As a generalization of the totally nonnegative matrices we further consider *sign regular* matrices, i.e., matrices with the property that all their minors of fixed order have one specified sign or are allowed also to vanish. We identify some subclasses of the sign regular matrices which exhibit the interval property. The subclasses which require to check only two vertex matrices include the following sets (here it is understood that the two bound matrices have the same signature of their minors):

- the strictly sign regular matrices, i.e., the matrices with the property that all their minors of fixed order have one (strict) specified sign;
- the nonsingular almost strictly sign regular matrices, a class in between the nonsingular sign regular matrices and the strictly sign regular matrices;

- the tridiagonal nonsingular sign regular matrices;
- the nonsingular totally nonpositive matrices, i.e., the matrices with the property that all their minors are nonpositive.

In some instances, the assumption of nonsingularity can be somewhat relaxed. These results lead us to the following new conjecture: Assume that the two bound matrices with respect to the checkerboard ordering are nonsingular and sign regular; then all matrices lying between the two bound matrices are nonsingular and sign regular, too. It was shown in [Garloff, 1996] that the conclusion is true if we consider instead of the two bound matrices a set of vertex matrices with the cardinality of at most 2^{2n-1} (n being the order of the matrices).

Keywords: interval matrix, Checkerboard ordering, totally nonnegative matrix, sign regular matrix.

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Isometries of Grassmann spaces

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Abstract

Let $\mathcal{H}$ be a complex Hilbert space and $\mathcal{P}_1$ denote the set of rank-one and self-adjoint projections on $\mathcal{H}$. The celebrated Wigner theorem characterizes those transformations of $\mathcal{P}_1$ that preserve the so-called transition probability. Equivalently, this theorem describes the isometries of $\mathcal{P}_1$ with respect to the operator norm. Namely, these transformations come from linear and conjugate linear isometries of the underlying Hilbert space $\mathcal{H}$.

The following is a natural question: how can we generalize this famous and highly important theorem? One way is to consider the space of rank-n, self-adjoint projections on $\mathcal{H}$ – denoted by $\mathcal{P}_n(\mathcal{H})$ and referred to as the Grassmann space – and characterize the isometries with respect to some norm, e. g., the operator norm.

In my talk I would like to show some recent results concerning this direction.

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The generation of all rational orthogonal matrices in $R^{p,q}$

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Abstract

A method for generating all the rational orthogonal matrices on generalized scalar product spaces, $R^{p,q}$, is presented. The proposed method is based on the proof of a weak version of the Cartan-Dieudonné theorem, handled using Clifford algebras.

Keywords: orthogonal matrices, Cartan-Dieudonné, Householder transformations, Clifford algebras.

Linear systems with singular $g-$circulant matrices

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Abstract

The application of generalized inverses to the study of consistent and inconsistent linear systems of equations is well known from the literature. The aim of this research is to characterize solutions in case of linear systems with singular $g-$circulants of order n, defined as $C_{n,g} = [a_{(r-gs)modn}]_{r,s=0}^{n-1}$.

We begin with a characterization of classical generalized inverses of this class of matrices over the complexes, based on specific factorizations such as

$$C_n = F_n D_n F^*{}_n, \quad C_{n,g} = C_n Z_{n,g},$$

where F_n is the Fourier matrix, $*$ is the involution $(a_{ij}) \to (\overline{a}_{ij})^T$, and

$$Z_{n,g} = [\delta_{r-gs}]_{r,s=0}^{n-1}, \quad \delta_k = \begin{cases} 1 \text{ if } k \equiv 0(\text{mod} n) \\ 0 \text{ otherwise} \end{cases} .$$

A study of generalized inverses of singular g-circulants over domains is also presented.

Keywords: g-circulant, Toeplitz-related matrices, generalized inverses.

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Ordering matrices with some nonnegativity properties

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Abstract

In this paper we consider some structured matrices having some property of nonnegativity. Specifically, for matrices of index at most 1, we are going to consider those whose group projector is nonnegative. We will show how the structure on a given successor is inherited by their predecessors under the minus and sharp partial orderings.

This paper has been partially supported by the DGI project with number MTM2013-43678-P.

Keywords: partial ordering, nonnegative matrices, minus ordering, group projector.

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On relation between $P-$matrices and regularity of interval matrices

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Abstract

A P−matrix is a matrix with positive principal minors, and this concept plays an important role in proving existence and uniqueness of linear complementarity problems. An interval matrix is a set of matrices that entry-wise lie between two given matrices. An interval matrix is called regular if it contains only nonsingular matrices.

Checking whether a given matrix is a $P-$matrix is an $NP-$hard problem, as well as checking regularity of an interval matrix. There are known interesting relations between these two properties; see References.

We explore new results connecting both properties. In particular, we show that an interval matrix is regular in and only if some special matrices constructed from its center and radius matrices are $P-$matrices. We also investigate the converse direction. We reduce the problem of checking $P-$matrix property to regularity of a special interval matrix. Based on this reduction, novel sufficient condition for a $P-$matrix property is derived, and its strength is inspected.

Keywords: $P-$matrix, interval matrix, nonsingularity.

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Banded matrices with banded inverses and parallel computations

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Abstract

In the talk we give a survey of recent results on banded matrices with banded inverses. All such matrices are products of block diagonal matrices (usually with different sizes of blocks). The number of factors is controlled by the bandwidth and not by a size of a matrix. We survey results on this problem. The authors found a factorization of triangular matrices by two block diagonal matrices. Fast algorithm for parallel computations which uses this factorization is described.

Important banded matrices with banded inverses arise in constructing orthogonal polynomials on the unit circle, they also yield as filter banks with perfect reconstruction, the key to wavelets (Toeplitz matrices and CMV-matrices).

Keywords: linear system, triangular matrix, banded matrix, factorization, parallel computing.

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Reduced differential transform method for nonlinear KdV type equations

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Abstract

Three models of KdV equation called KdV, K(2, 2) and modified KdV as given respectively by

$$u_t - 3\left(u^2\right)_x + u_{xxx} = 0$$
$$u_t + \left(u^2\right)_x + \left(u^2\right)_{xxx} = 0$$
$$u_t + \frac{1}{2}\left(u^2\right)_x - u_{xxx} = 0$$

KdV Equations are the important equations that gives rise to solitary wave solutions. Solitons, which are waves with infinite support, are generated as a result of the balance between the nonlinear convection $\left(u^n\right)_x$ and the linear dispersion u_{xxx} in the above equations.

In this paper, a general framework of the reduced differential transform method is presented for solving the nonlinear KdV type equations. The method is extremely simple and concise, and comparison with the Variational Iteration Method and Homotopy Perturbation Method reveals that the present method is an attracting mathematical tool.

Keywords: KdV equation, solitary wave solutions, reduced differential transform method.

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A meshless method of lines with Lagrange interpolation polynomials for the numerical solutions of Burger's Fisher equation

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Abstract

Recently, meshless method of lines using Radial basis functions (RBFs) has been used to solve Burger's Fisher equation. In this work, an alternative approach called the meshless method of lines using lagrange interpolation polynomials present to overcome demerit of interpolation matrix calculation meshless method of lines using RBFs. To test the accuracy of this new method, L_2 and L_∞ error norms are calculated for each test problems. Comparing the methodology with some known techniques shows that the present approach is effective, practice and powerful.

Keywords: Meshless method of lines, Burger's Fisher equation, Lagrange interpolation polynomials.

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Notes on some spectral radius and numerical radius inequalities

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Abstract

We prove numerical radius inequalities for products, commutators, anticommutators and sums of Hilbert space operators. A spectral radius inequality for sums of commuting operators is also given. Our results improve earlier well-known results.

A graphical-based approach for fault detection in a cement rotary kiln system

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Abstract

This work introduces a graph-based approach to the representation of a large sized multivariate data used for monitoring of a cement rotary kiln system. In this paper, we suggest to use a two dimensional plot characterizing the statistical variability of a large sized multivariate data. This graphical representation is based on the mean and variance values. These two statistical parameters plot is used to assess perfectly the fault detection in a cement rotary kiln system. The reliability and accuracy are the consequence of the developed adaptive threshold. This threshold is obtained through several repeated experiments under the healthy mode and the same operating conditions of the system. An appropriate statistical test is used to examine the validity of the adaptive threshold estimation approach. Furthermore, at each meanŠs subinterval and for all experiments a confidence interval is obtained which is closely linked to the distribution frequencies of the variance as a random variable. In addition, several significance levels are considered to show the performances of the proposed adaptive thresholding technique compared to the limitations of the fixed threshold through the rate of false alarms. It is demonstrated from various experimental faulty mode of a cement rotary kiln system the effectiveness and accuracy of the adaptive threshold in terms of no false alarms and negligible missed alarms contrarily to the fixed threshold.

Keywords: fault detection, adaptive threshold, large sized multivariate data, mean, variance, statistical test, confidence interval, significance level, cement rotary kiln.

Efficiency of the improved estimators with stochastic restrictions under balanced loss in linear regression models

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Abstract

This paper studies and compares the performance properties of weighted average estimators of ordinary least squares and improved estimators by considering balanced loss function proposed by Zellner (1994). Superiority conditions have been derived, assuming error distribution to be non-normal.

Keywords: linear regression model, stochastic linear restrictions, ordinary least squares estimator, mixed regression estimator, improved estimator, weighted average estimators, balanced loss function.

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On the lengths of generating sets of matrix algebras

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Abstract

Let $\mathbf{F}$ be a field and $\mathbf{A}$ a subalgebra of the full matrix algebra $\mathrm{M}_n(\mathbf{F})$. The length, $\ell(\mathbf{S})$, of a generating set $\mathbf{S}$ of $\mathbf{A}$, is the least nonnegative integer m for which $\mathbf{A}$ is spanned by the monomials in the elements of $\mathbf{S}$ of degree at most m, and the length, $\ell(\mathbf{A})$, of $\mathbf{A}$ is the maximum of the set $\{\ell(\mathbf{S})\colon \mathbf{S}$ generates $\mathbf{A}\}$. We compute $\ell(\mathbf{A})$ for various subalgebras of $\mathrm{M}_n(\mathbf{F})$. Azaria Paz [Paz, 1984] made the still unresolved conjecture that $\ell(M_n(\mathbf{F})) = 2n-2$ and we discuss the current state of research on this topic.

We present a number of generating sets $\mathbf{S}$ of $\mathrm{M}_n(\mathbf{F})$ with $\ell(\mathbf{S}) = 2n-2$. In particular, we show that if j is an integer with $1 \leq j < n$ and $\gcd(j,n) = 1$, and J the full nilpotent $n \times n$ Jordan block, then $\mathbf{S} = \{J^j, (J^T)^{n-j}\}$ generates $\mathrm{M}_n(\mathbf{F})$ and satisfies $\ell(\mathbf{S}) = 2n-2$.

Keywords: matrix algebra, generating sets, length.

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Accurate computation of the pseudoinverse of strictly totally positive matrices

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Abstract

The problem of computing the pseudoinverse of strictly totally positive matrices, that is, matrices with all its minors positive, is considered. Starting from the accurate bidiagonal decomposition of a given strictly totally positive matrix A, our algorithm computes its pseudoinverse A^+ by using the accurate algorithm for computing the QR factorization of a nonsingular totally positive matrix due to P. Koev [Koev, 2007] and the new algorithm we have developed for the accurate computation of the inverse of a strictly totally positive matrix. Although our approach takes into account the explicit expression of A^+, $A^+ = (A^T A)^{-1} A^T$, it is shown that the computation of matrix A is not necessary. Numerical experiments for strictly totally positive Vandermonde and Bernstein-Vandermonde matrices [Marco and Martínez, 2013] which show the good behaviour of our algorithm are also included.

Keywords: pseudoinverse, inverse, totally positive matrix, bidiagonal decomposition, high relative accuracy.

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Matrix methods in sufficiency problem

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Abstract

The notion of quadratic sufficiency was introduced and characterized in [Mueller, 1987] in the context of fixed linear model. It was proved there that under normality quadraticly sufficient statistic is also sufficient. In the paper it is studied the problem of estimation in possibly misspecified linear model; i.e. when some effects assumed to be fixed are random. It is shown that quadraticly sufficient statistic under fixed model it is sufficient under respective mixed linear normal model. The results are obtained using matrix methods including formula for generalized inverse of sum of matrices.

Keywords: generalized inverse, linear sufficiency, quadratic sufficiency, mixed model.

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About the number of characteristic subspaces

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Abstract

Given $A \in M_n(\mathbb{F})$ a nilpotent matrix and $\mathbb{F}$ an arbitrary field, a A-invariant subspace is called hyperinvariant (respectively characteristic) if it is also B-invariant for all of the matrices B (respectively, nonsingular matrices B) commuting with A. Let us denote by $Hinv(A)$ and $Chinv(A)$ the lattices of hyperinvariant and characteristic subspaces, respectively. Obviously:

$$Hinv(A) \subseteq Chinv(A)$$

Moreover, $Chinv(A) = Hinv(A)$ if $\mathbb{F} \neq GF(2)$. For $\mathbb{F} = GF(2)$, the subspaces in $Chinv(A) \setminus Hinv(A)$ are characterized as direct sums $Z \oplus Y$, where Y, Z are subspaces associated to a so-called char-tuple. We compare the number of characteristic non-hyperinvariant subspaces and the number of hyperinvariant subspaces.

Keywords: hyperinvariant subspaces, characteristic subspaces, binomial Gaussian number.

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Graphs that allow all the eigenvalue multiplicities to be even

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Abstract

If G is an undirected graph on n vertices, let $S(G)$ be the set of all $n \times n$ real symmetric matrices whose nonzero off-diagonal entries occur in exactly the positions corresponding to the edges of G.

The Inverse Eigenvalue Problem for a graph G is a problem of determining all possible lists that can occur as the lists of eigenvalues of matrices in $S(G)$. This question is, in general, hard to answer. In this talk we discuss some of the related questions of characterizing possible multiplicities of eigenvalues of matrices in $S(G)$. In particular, we will be interested in determing graphs G such that there exists a matrix in $S(G)$ whose multiplicities of eigenvalues are all even.

Keywords: symmetric matrix, eigenvalue, maximum multiplicity, graph.

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Some inequalities and Schur complements of block Hadamard product

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Abstract

Let $A = (A_{ij})$ and $B = (B_{ij})$ be qxq block positive definite matrices in which each block is an $n \times n$ matrix with complex entries. Denote the block-Hadamard product of A and B by $A \odot B$ which was defined by Horn, Mathias, and Nakamura. We first prove some inequalities for the inverse of block Hadamard product of two block commuting positive definite matrices. For any C and D of size $q \times q$ block matrices under strong commutation assumtions

$$(C \odot D)(A \odot B)^{-1}(C \odot D)^* \leq (CA^{-1}C^*) \odot (DB^{-1}D^*)$$

In particular

$$(A \odot B)^{-1} \leq A^{-1} \odot B^{-1}$$

Then we give three inequalities which are releated to the Schur complements of block Hadamard product $A \odot B$ and its inverse $(A \odot B)^{-1}$ as

$$(A \odot B)/\alpha \geq A/\alpha \odot B/\alpha,$$

$$(A \odot B)^{-1}/\alpha \leq [(A \odot B)/\alpha]^{-1} \leq (A/\alpha)^{-1} \odot (B/\alpha)^{-1},$$

and

$$(A \odot B)^{-1}/\alpha \leq \left(A^{-1}/\alpha\right) \odot \left(B^{-1}/\alpha\right) \leq (A/\alpha)^{-1} \odot (B/\alpha)^{-1}.$$

Keywords: block Hadamard product, Schur complements, matrix inequalities.

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The generalized quadraticity of linear combination of two commuting quadratic matrices

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Abstract

Let A_1 and A_2 be two nonzero $\{\alpha_1, \beta_1\}-$quadratic matrix and $\{\alpha_2, \beta_2\}-$quadratic matrix, respectively with $\alpha_1 \neq \beta_1$ and $\alpha_2 \neq \beta_2$. The aim of this work is mainly to characterize all situations, where the linear combination $A_3 = a_1A_1 + a_2A_2$ is a generalized quadratic matrix. The results established in here cover many of the results in the literature related to idempotency, involutivity, and tripotency of the linear combinations of idempotent and/or involutive matrices. Finally, some numerical examples are given to exemplify the main result.

Keywords: quadratic matrix, generalized quadratic matrix, linear combination, idempotent matrix.

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On the relative linear sufficiency

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Abstract

In this talk we consider the concept of relative linear sufficiency of statistics Fy when estimating the estimable parametric function of β under the linear model $A = \{y, X\beta, V\}$. The concept of linear sufficiency was essentially introduced in early 1980s by Baksalary, Kala and Drygas, but to our knowledge the concept of relative linear sufficiency has not appeared in the literature. In this talk we consider some possibilities to measure the relative linear sufficiency.

Keywords: best linear unbiased estimator, linear model, linear sufficiency, transformed linear model.

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On orthogonal integrators for isospectral flows

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Abstract

We study several numerical methods to solve the matrix differential system $L' = B(L)L - LB(L)$, where L and $B(L)$ are symmetric and skew--symmetric matrices, respectively (such systems arise in the so-called isospectral flows). Although the magnitude of the error in the computed approximations is always dependent on the order of the integrator used, the spectrum and the symmetry of the exact solutions may be preserved, to working accuracy, if appropriate methods are chosen. We review the relevant literature where two different classes of methods have been proposed: automatic orthogonal integrators and projected automatic integrators. The last ones are to be preferred on the ground of computational efficiency. In previous works, two orthogonal projections have been proposed in the context of the integration of orthogonal flows (these are closely related to isospectral flows): those that replace a time-step approximation, a matrix already close to orthogonal, by the orthogonal factor computed either with the QR decomposition or the polar decomposition. We show that for medium/large size systems, the QR approach is more efficient. A lot of attention is paid to prove that the QR based projected methods produce high quality solutions. Results of several numerical experiments are given and explained.

Keywords: matrix differential equations, Taylor's methods, orthogonal projections.

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An alternative canonical form for $H-$orthogonal matrices

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Abstract

For an invertible real symmetric matrix H we define the class of $H-$expansive matrices as those matrices A for which $A^THA - H \geq 0$. A matrix A is H-orthogonal if $A^THA - H = 0$. It is therefore possible in this case to simplify the simple form for $H-$expansive matrices even further by choosing suitable Jordan bases. This novel approach leads to more transparent formulas for H, starting with A in Jordan canonical form.

Keywords: $H-$orthogonal matrices, $H-$expansive matrices, canonical forms.

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Iterative method for linear system with coefficient matrix as an $M_\vee-$matrix

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Abstract

An iterative technique to solve linear system $Ax = b$ involves with an initial approximation x_0 to the solution x and determines a sequence $\{x_k\}$ that converges to the exact solution x. Most of these methods reduce to the iterative scheme $x^{k+1} = Hx^k + c$, with $k \geq 0$. As it is well known that with a spitting $A = M - N$ of A, one may associate an iterative scheme $x^{k+1} = M^{-1}Nx^k + M^{-1}b$, for solving the system and the convergence of such iterative scheme depends on the spectral radius of $M^{-1}N$. In this paper we study iterative procedures associated with a splitting of A, to solve the linear system $Ax = b$, with the coefficient matrix A an $M_\vee$-matrix. We generalize the concepts of regular and weak regular splitting of a matrix using the notion of eventually nonnegative matrix, and term them as E-regular and weak E-regular splitting respectively. We develop necessary and sufficient condition for the convergence of these type of splittings. We also discuss the convergence of Jacobi and Gauss-Seidel spittings for $M_\vee$-matrices.

The following concepts are introduced in this paper.

Definition 1: For $A \in \mathbb{R}^{n,n}$, a splitting $A = M - N$ is said to be an $E-$regular splitting if both M^{-1} and N are nonnilpotent eventually nonnegative matrices.

Definition 2: For $A \in \mathbb{R}^{n,n}$, a splitting $A = M - N$ is said to be a weak $E-$regular splitting if both $M^{-1}N$ and M^{-1} are nonnilpotent eventually nonnegative matrices.

Keywords: eventually nonnegative, regular splitting, weak regular splitting, Jacobi method, Gauss-Seidel method.

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A wider convergence area for the MSTMAOR iteration methods for LCP

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Abstract

In order to solve large sparse linear complementarity problems on parallel multiprocessor systems, modulus-based synchronous two-stage multisplitting iteration methods based on two-stage multisplittings of the system matrices were constructed and investigated in [Bai and Zhang, 2013b]. These iteration methods include the multisplitting relaxation methods such as Jacobi, Gauss-Seidel, SOR and AOR of the modulus type as special cases. In the same paper the convergence theory of these methods is developed, under the following assumptions: (i) the system matrix is an H+-matrix and (ii) one acceleration parameter is greater than the other. Here we show that the second assumption can be avoided, thus enabling us to obtain an improved convergence area. The result is obtained using the similar technique proposed in [Cvetković and Kostić, 2014] and its usage is demonstrated by an example of the LCP.

Keywords: relaxation method, linear complementarity problem, multisplitting, H-matrices.

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Centrohermitian and skew-centrohermitian solutions to a pair of quaternion matrix equations $(AXB, DXE) = (C, F)$

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Abstract

Suppose that the quaternion matrix equations $AXB = C$, $DXE = F$ are given, where X is an unknown quaternion matrix and A, B, C, D, E, and F are known quaternion matrices of suitable size. In this paper, the explicit expression of the best approximate solution of matrix nearness problems over the set of centrohermitian, and skew-centrohermitian matrices are established for this system of quaternion matrix equations by using Moore–Penrose Inverse, the Kronecker product, and the complex representations of quaternion matrices. Moreover, a numerical algorithm is added for finding the solutions of the problems considered at the end of the study.

Keywords: best approximate solution, quaternion matrix equations, matrix nearness problem, the minimum residual problem, Moore–Penrose inverse.

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A short survey of the Sanov Problem and its alterations

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Abstract

Let
$$A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad B_\lambda = \begin{bmatrix} 1 & 0 \\ \lambda & 1 \end{bmatrix}.$$
We call complex number, λ, "free" if the group generated by A and B_λ is free and "semigroup free" if the semigroup generated by A and B_λ is free. The problem "which complex numbers are free" is assigned to I. N. Sanov and is almost 70 years old.

We present a survey of known free and semigroup free λ's and we also relate them with some generalized Fibonacci polynomials.

Keywords: free groups of matrices, free semigroups of matrices, Fibonacci polynomials.

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Idempotent preservers of infinite matrices

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Abstract

The topic of the talk is describing the maps ϕ defined on the algebra of $\mathbb{N}\times\mathbb{N}$ triangular matrices over fields F of characteristic different from 2, that have the following property: $x-\lambda y$ is idempotent if and only if $\phi(x)-\lambda\phi(y)$ is idempotent for all $\lambda \in F$. We will present a theorem that states that every such map is some sort of a sum of compositions of some standard maps. We will also show that this result can be used when considering the form of the maps that preserve inverses of the infinite triangular matrices.

Keywords: preserver, idempotent, infinite triangular matrix, invertibility preservers.

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Positive coefficients of power series related to the spectral gap

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Abstract

The problem of deciding whether a given function has a power series expansion with all its coefficients positive is of seemingly elementary nature, but can be surprisingly difficult, and only a few general families of functions with positive coefficients are known. We will discuss a family of functions with positive coefficients that arise from the characteristic polynomials of positive matrices. In particular, we show that for any positive matrix A there exists α_0 so that the power series expansion of

$$1 - \det(I_n - tA)^{\alpha}$$

has positive coefficients for all $\alpha \in (0, \alpha_0)$. We will explain how this result depends on the properties of a special nonnegative matrix arising in the study of the nonnegative inverse eigenvalue problem, and how this theory extends to some classes of classic orthogonal polynomials. We will also discuss the connection between positivity of coefficients of certain power series and the spectral gap of the associated nonnegative matrix.

Keywords: nonnegative matrix, inverse eigenvalue problem, positive coefficients, power series, orthogonal polynomials.

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Laffey, T., Loewy, R. and Šmigoc, H. Power series with positive coefficients arising from the characteristic polynomials of positive matrices. *Mathematische Annalen*, to appear.

Transformations preserving norms of means of positive operators

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Abstract

Motivated by recent investigations on norm-additive and spectrally multiplicative maps on various spaces of functions, in this presentation we determine all bijective transformations between the positive cones of standard operator algebras over a Hilbert space which preserve a given symmetric norm of a given mean of elements. (We note that by a standard operator algebra we mean a subalgebra of $B(H)$ the algebra of all bounded linear operators on H which contains all finite rank operators in $B(H)$. Furthermore, we say that the norm N on $B(H)$ is a symmetric norm, if $N(AXB) \leq \|A\| N(X) \|B\|$ holds for all $A, B, X \in B(H)$.)

Keywords: preservers, operator means, symmetric norms.

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A matricial description of improved computation of the Bernstein coefficients and a convexity test for polynomials

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Abstract

Solving optimization problems is of paramount importance in many real-life and scientific problems; polynomial global optimization problems form a significant part of them. One approach for their solution is based on the expansion of a polynomial into Bernstein polynomials, the so-called *Bernstein form*, see [Cargo and Shisha, 1966], [Garloff, 1986], [Garloff, 1993], [Nataraj and Arounassalame 2007], [Ray and Nataraj 2009], [Zettler and Garloff 1998]. This approach has the advantage that it does not require function evaluations which might be costly if the degree of the polynomial is high.

Shorthand notation for multi-indices is used; a multi-index $(i_1, \ldots, i_n)$ is abbreviated as i, where n is the number of variables. Comparison between and arithmetic operations with multi-indices are defined entry-wise. For $x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n$, its monomials are defined as $x^i := \prod_{j=1}^n x_j^{i_j}$, and the abbreviations $\sum_{i=0}^k := \sum_{i_1=0}^{k_1} \ldots \sum_{i_n=0}^{k_n}$ and $\binom{k}{i} := \prod_{\alpha=1}^n \binom{k_\alpha}{i_\alpha}$ are used.

We will consider the unit box $\mathbf{u} := [0,1]^n$, since any compact nonempty box $\mathbf{x}$ of $\mathbb{R}^n$ can be mapped affinely upon $\mathbf{u}$. Let p be an n-variate polynomial of degree l which can be represented in the power form as $p(x) = \sum_{i=0}^l a_i x^i$. We expand p into Bernstein polynomials over $\mathbf{u}$ as

$$p(x) = \sum_{i=0}^{k} b_i^{(k)} B_i^{(k)}(x), \quad k \geq l, \tag{1}$$

where $B_i^{(k)}$ is the *i-th Bernstein polynomial of degree k*, $k \geq l$, defined as

$$B_i^{(k)}(x) = \binom{k}{i} x^i (1-x)^{k-i}. \tag{2}$$

The coefficients of this expansion are called the *Bernstein coefficients* of p over $\mathbf{u}$ and are given by ($a_j := 0$ for $j > l$)

$$b_i^{(k)} = \sum_{j=0}^{i} \frac{\binom{i}{j}}{\binom{k}{j}} a_j, \quad 0 \leq i \leq k. \tag{3}$$

The Bernstein coefficients can be organized in a multi-dimensional array $B(\mathbf{u}) = (b_i^{(k)})_{0 \le i \le k}$, the so-called *Bernstein patch*.

The Bernstein coefficients provide lower and upper bounds for the range of $p(x)$ over $\mathbf{u}$,

$$\min b_i^{(k)} \le p(x) \le \max b_i^{(k)}, \quad \text{for all } x \in \mathbf{u}. \tag{4}$$

Equality holds in the left or right inequality in (4) if and only if the minimum or the maximum, respectively, is attained at a vertex of $\mathbf{u}$, i.e., if $i_j \in \{0, k_j\}$, $j = 1, \ldots, n$.

We can improve the enclosure for the range of p given by (4) by elevating the degree k of the Bernstein expansion or by subdividing $\mathbf{u}$. The subdivision is more efficient than the degree elevation.

From the Bernstein coefficients $b_i^{(k)}$ of p over $\mathbf{u}$, we can compute by the de Casteljau algorithm the Bernstein coefficients over sub-boxes $\mathbf{u}_1$ and $\mathbf{u}_2$ resulting from subdividing $\mathbf{u}$ in the s-th direction, i.e.,

$$\begin{aligned} \mathbf{u}_1 &:= [0,1] \times \ldots \times [0,\lambda] \times \ldots \times [0,1], \\ \mathbf{u}_2 &:= [0,1] \times \ldots \times [\lambda,1] \times \ldots \times [0,1], \end{aligned} \tag{5}$$

for some $\lambda \in (0,1)$.

Bounding the range of a function over a box is an important task in global optimization when a branch and bound approach is applied. In the case that the optimization problem is convex we have the advantage that each local minimum is also a global one. Therefore, it is useful to know when a function is convex over a box. A well-known criterion for convexity is that the Hessian matrix is positive definite.

In our talk we present the following results:

- We propose a new method for the computation of the Bernstein coefficients of multivariate Bernstein polynomials which involves matrix operations such as multiplication and transposition and which is more efficient than the matrix method presented in [Ray and Nataraj, 2012].
- We present a new method for the calculation of the Bernstein coefficients over a sub-box by premultiplying the matrix representing the Bernstein patch by matrices which depends on the intersection point λ.
- As an application to global optimization, we propose a test for the convexity of a polynomial p. This check employs the *interval Hessian matrix* that is obtained by the entry-wise application of the range enclosure property (4). Following [Rohn, 1994], we test the positive semidefiniteness of this interval matrix which leads to the test for convexity of p.

Keywords: Bernstein polynomials, Bernstein coefficients, range enclosure, subdivision, interval Hessian, convexity test.

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Semi-active damping optimization using the parametric dominant pole algorithm

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Abstract

We consider the problem of determining an optimal semi-active damping of vibrating systems. For this damping optimization we use a minimization criterion based on the impulse response energy of the system. The optimization approach yields a large number of Lyapunov equations which have to be solved, thus we propose an optimization approach that works with reduced systems which accelerate optimization process. Reduced systems are generated using the parametric dominant pole algorithm. The optimization process is additionally accelerated with a modal approach while the initial parameters for the parametric dominant pole algorithm are chosen during optimization procedure using residual bounds. Our approach calculates a satisfactory approximation of the impulse response energy while providing a significant acceleration of the optimization process. Numerical results illustrate the effectiveness of the proposed algorithm.

Keywords: semi-active damping, dominant poles, vibrational systems.

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On the convexity of Heinz means and unitarily invariant matrix norms

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Abstract

Let A and B be positive operators, X be a general operator on a complex separable Hilbert space, and $v \in [0,1]$. The operator version of Heinz inequalities is given by

$$2\left|\left|\left|A^{1/2}XB^{1/2}\right|\right|\right| \leq \left|\left|\left|A^{v}XB^{1-v}+A^{1-v}XB^{v}\right|\right|\right| \leq |||AX+XB|||$$

for every unitarily invariant norm $|||\cdot|||$. In this paper, utilizing the convexity of the function $f(v) = \left|\left|\left|A^{v}XB^{1-v}+A^{1-v}XB^{v}\right|\right|\right|$, $0 \leq v \leq 1$, and the well known Hermite-Hadamard inequality for the convex functions, we give new refinements of Hermite-Hadamard inequality which assert the new norm inequalities for the matrices.

Keywords: convex function, Hermite-Hadamard inequality, Heinz inequality, unitarily invariant norm.

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Special variance structures and orthogonal transformations

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Abstract

Using special variance structure in multivariate linear models prevents using standard statistical methods assuming general completely unknown variance matrix. This problem is for many years being solved via orthogonal transformations which diagonalize (or block-diagonalize) the variance matrix of the transformed vector. Some surprising properties of this methodology will be presented.

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Keywords: special variance structures, diagonalization, orthogonal transformations.

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Boundary value problem for second-order differential operators with integral boundary conditions

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Abstract

In this work, we study a second order differential operator with variable coefficients and weighted integral boundary conditions. Under certain conditions on the weighting functions, called non regular boundary conditions, we prove that the resolvent decreases with respect to the spectral parameter in $L^p(0,1)$, but there is no maximal decreasing at infinity for $p \geq 1$. Furthermore, the studied operator generates in $L^p(0,1)$ an analytic semi group with singularities for $p \geq 1$. The obtained results are then used to show the correct solvability of a mixed problem for a parabolic partial differential equation with non regular integral boundary conditions.

Keywords: Green's function, integral boundary conditions, non regular boundary conditions, semigroup with singularities.

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Part VIII

Contributed Poster

Knowledge about pediatric high blood pressure

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Abstract

The arterial hypertension (HTA) may emerge silently in childhood. HTA is diagnosed by regular measurement of blood pressure, which varies with age, sex and height and it can progress to adulthood. HTA is potentially associated to a severe organ damage, so knowing about the existence of hypertension and early intervention are important issues.

It is intended to identify if caregivers of children and young people have some knowledge about high blood pressure in childhood. Once a questionnaire can be very informative instrument when properly designed, an experimental questionnaire was applied to caregivers (first-degree relatives or their legal representatives) of children aged between 3 and 18 years. We applied a questionnaire to a sample of caregivers of children and/or adolescents, users of the National Health System, which attended external consultation of Santa Maria Hospital (general pediatrics and/or sub-specialties) for reasons not related to changes in the blood pressure.

After a preliminary data analysis, there are searched possible associations between socio-demographic variables (such as age, sex, race, place of residence, education level, occupation, etc.) and the demonstrated knowledge.

Some models involving the age, the level of education, the profession, the knowledge of HTA which silently may arise in childhood and the existence of risk factors are estimated. The evaluation about the age at which the blood pressure of children is also another significant fact. This work is a preliminary study using an experimental questionnaire. Generalized linear models and factor analysis were applied.

The redesign of the survey, the use of other techniques and conducting future studies can be performed in order to assess the possibility of improving the respective knowledge about HTA.

Keywords: hypertension, children, knowledge, caregivers, generalized linear models, factor analysis.

Acknowledgments

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Useful Coimbra Facts

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- Inhabitants: 99 200

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